PACIFIC JOURNAL OF MATHEMATICS
Vol. 133, No. 1, 1988

ON AN ANALOGUE OF THE WIENER TAUBERIAN THEOREM FOR SYMMETRIC SPACES OF THE NON-COMPACT TYPE

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Let X be a symmetric space and G the connected component of the group of isometries of X. If $f \in L^1(X)$, we consider conditions under which $\mathrm{Sp}\{{}^g f\colon g \in G\}$ is dense in $L^1(X)$ in terms of the "Fourier transform" of f. This continues earlier work on this kind of problem by L. Ehrenpreis and F. I. Mautner, R. Krier and the author.

1. Introduction. Let f be an integrable function on R. Then we have the famous Wiener-Tauberian theorem: If the Fourier transform $\hat{f}$ is a nowhere vanishing function on R, then the ideal generated by f is dense in $L^1(R)$. In **[EM1]** Ehrenpreis and Mautner observed that the exact analogue of the above theorem is no longer true if one considers the commutative Banach algebra of K-bi-invariant L^1-functions on a non-compact semi-simple Lie group G, where K is a maximal compact subgroup of G—i.e. Let $I_1(G)$ denote the commutative Banach algebra of K-bi-invariant L^1-functions on G. For $f \in I_1(G)$ let $\hat{f}$ denote its spherical Fourier transform (see §2). Then there exist functions $f \in I_1(G)$ such that $\hat{f}$ is nowhere vanishing on the maximal ideal space M of $I_1(G)$ and yet the algebra generated by f is *not* dense in $I_1(G)$. However when $G = \mathrm{SL}(2, R)$ they were able to show that a modified version of Wiener's theorem is true i.e. $\hat{f}$ nowhere vanishing on M *together* with the condition that "it does not go to zero too fast at ∞" would indeed imply that the ideal generated by f is dense in $I_1(G)$. (Theorems 6 and 7 of **[EM1]**.) This kind of result has been generalized by R. Krier **[K1]** when G is a non-compact connected semi-simple Lie group of real rank 1 and by the author for G of arbitrary rank **[Si]**.

However the problem becomes considerably more difficult if one asks the following question: Let $f_1, f_2, \ldots, f_n$ be functions in $I_1(G)$ such that their spherical Fourier transforms have no common zeros in M. If $|\hat{f}_1| + |\hat{f}_2| + \cdots + |\hat{f}_n|$ "does not go to zero too fast at ∞", then is the ideal generated by $f_1, \ldots, f_n$ dense in $I_1(G)$? In this paper we try to give an answer to this question when the rank of G is 1—The