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THE MAZUR PROPERTY FOR COMPACT SETS

ABDERRAZZAK SERSOURI

We give a "convex" characterization to the following smoothness property, denoted by (CI): every compact convex set is the intersection of balls containing it. This characterization is used to give a transfer theorem for property (CI). As an application we prove that the family of spaces which have an equivalent norm with property (CI) is stable under c_0 and l_p sums for $1 \leq p < \infty$. We also prove that if X has a transfinite Schauder basis, and Y has an equivalent norm with property (CI) then the space $X\hat{\otimes}_\rho Y$ has an equivalent norm with property (CI), for every tensor norm ρ.

Similar results are obtained for the usual Mazur property (I), that is, the family of spaces which have an equivalent norm with property (I) is stable under c_0 and l_p sums for $1 < p < \infty$.

Introduction. Mazur [6] was the first who considered the following separation property, denoted by (I):

Every bounded closed convex set is the intersection of balls containing it.

Later, Phelps [7] proved that property (I) is weaker than the Fréchet differentiability of the norm, and gave a dual characterization for (I) in the finite dimensional case.

Phelps' theorem was extended to the infinite dimensional case in [3], where the property (I) was dually characterized.

Here we will give another extension of Phelps' theorem by characterizing the following property, denoted by (CI):

Every compact convex set is an intersection of balls.

This property was recently introduced by Whitfield and Zizler [9].

We use this characterization to give a "transfer theorem" for property (CI), which is analogous to the one given for property (I) [2].

We also prove a stability result for property (CI), which is of the same nature as the one given by Zizler for l.u.c. renormings [10]. Our proof can be modified to give a similar stability result for property (I).

Some renorming results of Whitfield-Zizler [9], and Deville [2] are particular cases of these stability results.