PACIFIC JOURNAL OF MATHEMATICS
Vol. 133, No. 1, 1988

THE SELBERG TRACE FORMULA FOR GROUPS WITHOUT EISENSTEIN SERIES

PAUL F. RINGSETH

Let G be a reductive Lie group, Γ a nonuniform lattice in G. Let χ be a finite dimensional unitary representation of Γ. In order to have Eisenstein series, (G, Γ) must satisfy a certain assumption. The purpose of this note is to compute the Selberg trace formula for pairs (G, Γ) that do not possess Eisenstein series. A necessary preliminary to this, is a trace formula for $\operatorname{Ind}_\Gamma^G(\chi)$. This is also presented.

Introduction. Let G be a reductive Lie group of the Harish-Chandra class; let Γ be a nonuniform lattice in G. Let χ be a finite dimensional unitary representation of Γ. Denote by $L^2(G/\Gamma; \chi)$ the representation space of $\operatorname{Ind}_\Gamma^G(\chi)$—then G acts on $L^2(G/\Gamma; \chi)$ via the left regular representation $L_{G/\Gamma}$. Let $L_{G/\Gamma}^{\text{dis}}$ be the restriction of $L_{G/\Gamma}$ to $L_{\text{dis}}^2(G/\Gamma; \chi)$—the maximal completely reducible subspace. One of the central problems in the theory of automorphic forms is computing the trace of $L_{G/\Gamma}^{\text{dis}}(\alpha)$ $(\alpha \in C_c^\infty(G))$; viz. the Selberg trace formula.

Let $L_{\text{con}}^2(G/\Gamma; \chi)$ be the orthogonal complement of $L_{\text{dis}}^2(G/\Gamma; \chi)$ in $L^2(G/\Gamma; \chi)$ and let $L_{G/\Gamma}^{\text{con}}$ be the corresponding representation—then most attacks on the Selberg trace formula begin by expressing the integral kernel of $L_{G/\Gamma}^{\text{con}}(\alpha)$ $(\alpha \in C_c^\infty(G))$ in terms of Eisenstein series. However, a certain assumption (cf. p. 16 of **[L2]** and p. 62 of **[OW1]**) needs to be satisfied by the pair (G, Γ) in order for a satisfactory theory of Eisenstein series to exist. The purpose of this note is to compute the Selberg trace formula for pairs (G, Γ) without Eisenstein series; i.e. that do not satisfy the assumption supra.

In order to accomplish this a trace formula needs to be given for $L_{\text{dis}}^2(G/\Gamma; \chi)$, when $\chi \neq 1$. This has been done in the case $G = \mathrm{SL}_2(\mathbf{R})$ by Venkov (cf. **[V1]**). Moore has also done preliminary work for the real rank one situation (cf. **[M1]**). For the general case, Eisenstein series need to be defined with respect to χ and a spectral decomposition following Langlands needs to be given. This was accomplished by the author in his thesis (cf. **[R1]**).

When (G, Γ) does not possess Eisenstein series, the procedure to compute the trace formula is to describe $L_{G/\Gamma}$ in terms of the left