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UNITARY EQUIVALENCE OF INVARIANT SUBSPACES OF BERGMAN AND DIRICHLET SPACES

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In this paper we investigate unitary equivalence of invariant subspaces of the Bergman and the Dirichlet space. By definition, this means unitary equivalence of the multiplication operator M_z restricted to the invariant subspaces.

We show that no two invariant subspaces of the Bergman space are unitarily equivalent to one another unless they are equal. Under some extra assumption on the invariant subspaces we obtain a similar result for the Dirichlet space.

1. Introduction. Let T be a bounded linear operator on a Hilbert space $\mathscr{H}$. A closed subspace $\mathscr{M}$ of $\mathscr{H}$ is called invariant for T, if $T\mathscr{M} \subseteq \mathscr{M}$. The collection of all invariant subspaces of T is denoted by $\operatorname{Lat}(T, \mathscr{H})$.

We shall say that two invariant subspaces $\mathscr{M}, \mathscr{N} \in \operatorname{Lat}(T, \mathscr{H})$ are *unitarily equivalent* if the operators $T|\mathscr{M}$ and $T|\mathscr{N}$ are unitarily equivalent, i.e. if there is a unitary operator $U: \mathscr{M} \to \mathscr{N}$ such that $UT|\mathscr{M} = T|\mathscr{N} U$.

Some of the operators we are interested in arise as operators of multiplication by z on the spaces D_α, $\alpha \in \mathbf{R}$. These operators will in the following be denoted by (M_z, D_α). Recall that an analytic function f in the unit disc with Taylor series expansion $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$ is in D_α if and only if

$$\sum_{n=0}^{\infty} (n+1)^\alpha |\hat{f}(n)|^2 < \infty.$$

The norm on D_α is given by $\|f\|_\alpha^2 = \sum_{n=0}^{\infty} (n+1)^\alpha |\hat{f}(n)|^2$. For $\alpha = -1, 0$, or 1 one has the Bergman space L_a^2, the Hardy space H^2, or the Dirichlet space D, respectively.

It follows from Beurling's theorem that all invariant subspaces of the unilateral shift ($\alpha = 0$) are unitarily equivalent to one another. Unitary equivalence of invariant subspaces of H^2 of the polydisc was investigated by Agrawal, Clark, and Douglas, see **[1]**.

We shall consider two cases, where the situation drastically differs from the H^2 case. The first one deals with subnormal multiplication