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GROUPS OF ISOMETRIES OF A TREE AND THE KUNZE-STEIN PHENOMENON

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In this paper we prove that every group of isometries of a homogeneous or semihomogeneous tree which acts transitively on the boundary of the tree is a Kunze-Stein group. From this, we deduce a weak Kunze-Stein property for groups acting simply transitively on a tree (in particular free groups on finitely many generators).

1. Introduction. Let G be a locally compact group, then G is said to satisfy the "Kunze-Stein property" or sometimes G is called a "Kunze-Stein group" if $L^p(G) * L^2(G) \subset L^2(G)$ for every $1 < p < 2$.

This property was discovered by R. A. Kunze and E. M. Stein for the group $SL_2(\mathbf{R})$ **[15]**. Later the same property was proved for every connected semisimple Lie group with finite center by M. Cowling **[6]**. In this paper we prove that every locally compact group of isometries of a homogeneous or semihomogeneous tree has the Kunze-Stein property provided that G acts transitively on the boundary of the tree. The proof of our Theorem is based on M. Cowling's proof of the Kunze-Stein phenomenon for $SL_2(\mathbf{R})$ **[6]**. A weaker property is deduced for discrete groups acting simply transitively on the tree but not on the tree boundary.

It is known that the group $SL_2(\kappa)$, where κ is a local field, may be realized as a closed subgroup of the group of all isometries of a homogeneous tree in such a way that $SL_2(\kappa)$ acts transitively on the boundary **[17]**. In particular our result implies that $SL_2(\kappa)$ is a Kunze-Stein group for every local field. This was proved by Gulizia **[13]** for a local field κ such that the finite residue class field associated with κ is not of characteristic 2.

We follow the terminology and definitions of **[6]**. In particular $A(G)$ is the Fourier algebra of G as defined in [7]; $C_{00}(G)$ denotes the space of continuous functions with compact support and $L^p(G), 1 \leq p \leq \infty$, the usual L^p-space with respect to a fixed left Haar measure. As observed in **[6]**, a locally compact group G is a Kunze-Stein group if and only if $A(G) \subset L^q(G)$ for every $q > 2$. We will also use the theory of representations for groups acting on a tree developed by P. Cartier