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REPRESENTING HOMOLOGY CLASSES OF $C\mathbf{P}^2 \# \overline{C\mathbf{P}}^2$

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In this paper we determine the set of all second homology classes in $C\mathbf{P}^2 \# \overline{C\mathbf{P}}^2$ which can be represented by smoothly embedded two-spheres in $C\mathbf{P}^2 \# \overline{C\mathbf{P}}^2$.

We say a class $u \in H_2(M^4, \mathbf{Z})$ can be represented by S^2 if it can be represented by a smoothly embedded 2-sphere in M^4. The purpose of this note is to prove the following.

THEOREM. *Let η, ξ be canonical generators of $H_2(C\mathbf{P}^2 \# \overline{C\mathbf{P}}^2, \mathbf{Z})$. Then $\gamma = a\eta + b\zeta$, $a, b \in \mathbf{Z}$, can be represented by S^2 if and only if a, b satisfy one of the following conditions.*
(i) $||a| - |b|| \leq 1$, *or*
(ii) $(a, b) = (\pm 2, 0)$ *or* $(0, \pm 2)$.

REMARK 1. The "if" part of the theorem is known (see Wall [7], Mandelbaum [5, the proof of Theorem 6.6]).

REMARK 2. If $p \in \mathbf{Z}$, then $p\eta$ (or $p\xi$) is represented by S^2 if and only if $|p| \leq 2$ (see Rohlin [6]).

REMARK 3. If a, b are relatively prime integers, then $\gamma = a\eta + b\xi$ is realized by a topologically embedded locally flat 2-sphere by Freedman [2]. Hence smoothness condition in the theorem is essential.

By Remarks 1 and 2, the Theorem follows from the following.

PROPOSITION. *Let a and b be two integers satisfying*

$$(*) \qquad \begin{cases} \text{(i)} & ab \neq 0, \text{ and} \\ \text{(ii)} & ||a| - |b|| \geq 2. \end{cases}$$

Then $a\eta + b\xi$ is not represented by S^2.

Proof. Suppose conversely that $a\eta + b\xi$ is represented by S^2. By reversing orientation if necessary, we may assume $n = b^2 - a^2 > 0$. Let $M^4 = C\mathbf{P}^2 \# \overline{C\mathbf{P}}^2 \# (n-1)C\mathbf{P}^2$ with ξ_i's the generators of