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POSITIVE ANALYTIC CAPACITY BUT ZERO BUFFON NEEDLE PROBABILITY

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There exists a compact set of positive analytic capacity but zero Buffon needle probability.

1. Introduction. For a compact set E in the complex plane $\mathbf{C}$, $H^\infty(E^c)$ denotes the Banach space of bounded analytic functions outside E with supremum norm $\|\cdot\|_{H^\infty(E^c)}$. The analytic capacity of E is defined by

$$\gamma(E) = \sup\{|f'(\infty)|; f \in H^\infty(E^c),\ \|f\|_{H^\infty(E^c)} \le 1\},$$

where $f'(\infty) = \lim_{z\to\infty} z(f(z) - f(\infty))$ [**1**, p. 6]. Let $\mathscr{L}(r, \theta)$ ($r > 0$, $-\pi < \theta \le \pi$) denote the straight line defined by the equation $x\cos\theta + y\sin\theta = r$. The Buffon length of E is defined by

$$Bu(E) = \iint_{\{(r,\theta);\mathscr{L}(r,\theta)\cap E\neq\varnothing\}} dr\,d\theta.$$

Vitushkin [**7**] asked whether two classes of null-sets concerning $\gamma(\cdot)$ and $Bu(\cdot)$ are same or not (cf. [**2**], [**3**]). Mattila [**4**] showed that these two classes are different. (He showed that the class of null-sets concerning $Bu(\cdot)$ is not conformal invariant. Hence his method does not give the information about the implication between these two classes.) The second author [**5**] showed that, for any $0 < \varepsilon < 1$, there exists a compact set E_ε such that $\gamma(E_\varepsilon) = 1$, $Bu(E_\varepsilon) \le \varepsilon$. The purpose of this note is to show

THEOREM. *There exists a compact set E_0 such that $\gamma(E_0) = 1$, $Bu(E_0) = 0$.*

2. Cranks. To construct E_0, we begin by defining cranks. The 1-dimension Lebesgue measure is denoted by $|\cdot|$. For a finite union E of segments in $\mathbf{C}$, its length is also denoted by $|E|$. For $\rho > 0, z \in \mathbf{C}$ and a set $E \subset \mathbf{C}$, we write $[\rho E + z] = \{\rho\zeta + z; \zeta \in E\}$. With $0 \le \varphi < 1$ and a segment $J \subset \mathbf{C}$ parallel to the x-axis, we associate the closed segment $J(\varphi)$ of the same midpoint as J, parallel to the x-axis and of