PACIFIC JOURNAL OF MATHEMATICS
Vol. 133, No. 1, 1988

ABSTRACT RIEMANNIAN STRATIFICATIONS

GIACOMO MONTI BRAGADIN

Let A be an abstract stratification. Assume that every stratum X is a riemannian manifold. We first give some conditions, under which it is possible to endow A with a suitable metric extending a well known technique of riemannian manifolds. Then stronger conditions are introduced in order that the metric space A becomes a G-space. An abstract stratification with all the above conditions is called an abstract riemannian stratification. Whitney stratifications are, in a natural way, riemannian.

Introduction. An abstract stratification is the disjoint union of a locally finite set of smooth manifolds, the strata, glued together according to certain rules (see **[9]**, **[13]**, **[15]**, **[16]**).

The basic tools of differential calculus, smooth functions, vector fields and differential forms, can be also defined for an abstract stratification by means of "controlled" collections of "tools". Classical results can be extended to abstract stratifications (e.g. de Rham's theorems, see **[15]**, **[8]**) in this way.

On the other hand a controlled approach seems to be useless when we take into account a riemannian structure: control conditions on the metric of the strata are too strict (bundle-like metrics, see **[11]**, are not controlled in general).

The purpose of this paper is to obtain weaker conditions in order that riemannian metrics of the strata glue together in an appropriate manner. In §1 sufficient conditions are given for extending the fundamental theorem of the riemannian geometry; in §2 a definition of compatibility among the riemannian structures of the strata is given and connections with the structure of G-space are studied. It is shown that the axioms of finite compactness, convexity, local prolongability are satisfied in mild hypotheses, whereas local uniqueness and existence of geodesic coordinates must be required (the former is an open question even in manifolds with boundary, see **[1]**).

0. Notation. 1. By a manifold we shall always mean a smooth manifold; topologically it is a connected, paracompact space. For any