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ON AN EXTENSION OF THE IKEHARA TAUBERIAN THEOREM

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A specific example of the Ikehara Tauberian theorem is extended to the case where the zeta function has a pole of order $p > 1$ at the first singularity. And we have an application to asymptotic behavior of eigenvalues for some partial differential operator.

0. Introduction. In order to study the asymptotic behavior of eigenvalues for some differential or pseudodifferential operators, one frequently uses a specific example of Ikehara's Tauberian theorem. To be more precise, let P be a positive definite self-adjoint operator on a separable Hilbert space H with the domain of definition K which is dense in H. If we denote the spectral resolution associated to P by $\{E(\lambda)\}$, we can define complex powers of P:

$$P^z = \int_0^\infty \lambda^z \, dE(\lambda) \tag{0.1}$$

where λ^z for $\lambda > 0$ take the principal values. If we assume that the canonical injection from K which is equipped with the graph norm to H is compact, it is well known that the spectrum $\sigma(P)$ of P is discrete. This enables one to write the sequence of eigenvalues by $0 < \lambda_1 \leq \lambda_2 \leq \cdots$, $\lambda_k \to \infty$ $(k \to \infty)$ with repetition according to multiplicity and let $N(\lambda)$ be the counting function of eigenvalues: $N(\lambda) = \#\{j; \lambda_j \leq \lambda\}$. If $\sum_{j=1}^\infty \lambda_j^a$ is convergent for some $a < 0$, P^z is of trace class and for $\operatorname{Re} z < a$,

$$\operatorname{Tr} P^z = \sum_{j=1}^\infty \lambda_j^z.$$

Then a specific example of Ikehara's Tauberian theorem says:

PROPOSITION 1. (*Wiener* [**13**] *and Donoghue* [**5**].) *Let* $\operatorname{Tr} P^z$ *be holomorphic for* $\operatorname{Re} z < a$ (< 0). *Assume that there exists a constant A such that*

$$\operatorname{Tr} P^z - \frac{A}{z-a}$$