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FINITELY GENERATED ALGEBRAS AND ALGEBRAS OF SOLUTIONS TO PARTIAL DIFFERENTIAL EQUATIONS

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We consider two types of uniform algebras A on the closure $\bar{\Omega}$ of a domain $\Omega \subset \mathbf{R}^n$: those generated by finitely many smooth functions and those consisting of solutions to $Lu = 0$ where L is a smooth complex vector field on Ω. Under certain conditions we prove the existence of one of two types of analytic structure in the maximal ideal space M_A of such an algebra: local foliations of Ω by complex manifolds on which the functions in the algebra are holomorphic, or foliations of a subset of $M_A \backslash \bar{\Omega}$ by analytic disks. Some open questions suggested by this line of inquiry are discussed.

1. Introduction. It is a central problem in the theory of uniform algebras to uncover general hypotheses on a proper subalgebra A of $C(X)$ which imply the existence of analytic structure in the maximal ideal space M_A of A (see **[Br]** or **[G]**). The purpose of this paper is to exhibit under certain conditions such analytic structure in the maximal ideal spaces of the following two classes of algebras:

1. *Finitely generated algebras.* Suppose $f_1, \ldots, f_k$ are C^∞ functions in a neighborhood of the closure of a bounded, smoothly bounded domain $\Omega \subset \mathbf{R}^n$, and suppose that $f_1, \ldots, f_k$ separate points on $\bar{\Omega}$. Let $\mathbf{C}[f_1, \ldots, f_k]$ be the algebra of polynomials in $f_1, \ldots, f_k$, and let A be the closure of $\mathbf{C}[f_1, \ldots, f_k]$ in $C(\bar{\Omega})$. Then A is a uniform algebra on $\bar{\Omega}$. Assuming that A is nowhere locally dense in the continuous functions, and that $F = (f_1, \ldots, f_k)$ is an imbedding of $\bar{\Omega}$ into $\mathbf{C}^k$, then we obtain analytic structure in M_A (Theorem 3.2).

To study the algebra of polynomials in $f_1, \ldots, f_k$, we consider the map $F = (f_1, \ldots, f_k)$ of $\bar{\Omega}$ into $\mathbf{C}^k$, which induces an isomorphism of A with $P(K)$, the algebra of uniform limits of polynomials in $(z_1, \ldots, z_k)$ on $K = F(\bar{\Omega})$. If F is a diffeomorphism, then we can use well-known results concerning real submanifolds of $\mathbf{C}^k$ to study the algebra $P(K)$. These include a theorem of Hörmander and Wermer **[H-W]** on polynomial approximation on totally real submanifolds of $\mathbf{C}^n$, a theorem of Freeman on complex foliations of real submanifolds of $\mathbf{C}^n$, and a