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POTENTIAL ESTIMATES IN ORLICZ SPACES

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We study estimates of the form

$$M_0^{-1}\left(\int M_0(u)\,d\mu(x)\right) \le CM^{-1}\left(\int M([1-\Delta]^m u)\,d\nu(x)\right)$$

for $u(x) \in C^\infty(\mathbf{R}^n)$, where $M_0(t)$, $M(t)$ are convex functions and μ, ν are measures. We apply this inequality to the study of boundary value problems for quasilinear partial differential equations.

1. Introduction. In recent years there has been considerable interest in inequalities of the form

$$\left(\int |u(x)|^q\,d\mu(x)\right)^{1/q} \le C\left(\int |(1-\Delta)^m u|^p\,d\nu(x)\right)^{1/p}, \qquad u \in C^\infty(\mathbf{R}^n) \tag{1.1}$$

(cf. [3–24] and the references quoted in them). Such inequalities have widespread applications to both linear and non-linear problems. We outline one here. Suppose one is interested in solving the Dirichlet problem

$$(1-\Delta)^m u = f(x,u) \quad \text{in } \Omega \subset \mathbf{R}^n, \qquad u(x) = 0 \quad \text{on } \partial\Omega \tag{1.2}$$

(here $\partial\Omega$ is the boundary of the domain Ω). One can solve (1.2) by topological methods if one can show that

$$\int |(1-\Delta)^m u(x)|^p\,d\nu(x) < \infty \tag{1.3}$$

implies

$$\int |f(x,u(x))|^p\,d\nu(x) < \infty. \tag{1.4}$$

If $f(x,t)$ satisfies

$$|f(x,t)| \le \sum V_k(x)|t|^{\alpha_k} \tag{1.5}$$

and the inequalities

$$\left(\int V_k(x)^p |u(x)|^{\alpha_k p}\,d\nu(x)\right)^{1/\alpha_k p} \le C_k\left(\int |(1-\Delta)^m u(x)|^p\,d\nu(x)\right)^{1/p} \tag{1.6}$$