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INTERSECTION HOMOLOGY OF WEIGHTED PROJECTIVE SPACES AND PSEUDO-LENS SPACES

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In this paper we calculate the integral intersection homology groups of weighted projective spaces and pseudo-lens spaces. Most computations of intersection homology have been for the rational groups. The groups calculated here have interesting torsion.

1. Introduction. Let $b = (b_0, \ldots, b_n)$ be an $(n+1)$-tuple of positive integers. The weighted projective space is by definition

$$\mathbf{P}_n(b_0, \ldots, b_n) = \{(z_0, \ldots, z_n) \in \mathbf{C}^{n+1} - \{0\}\}/\sim$$

where $(z_0, \ldots, z_n) \sim (\lambda^{b_0} z_0, \ldots, \lambda^{b_n} z_n)$, $\lambda \in \mathbf{C}^\times = \mathbf{C} - \{0\}$.

Let $b' = (b_1, \ldots, b_n)$. The pseudo-lens space is by definition

$$L_n(b_0; b_1, \ldots, b_n) = \left\{ (z_1, \ldots, z_n) \in \mathbf{C}^n \,\middle|\, \sum_{i=1}^{n} |z_i|^2 = 1 \right\} / \sim$$

where $(z_1, \ldots, z_n) \sim (\zeta^{b_1} z_1, \ldots, \zeta^{b_n} z_n)$, $\zeta \in \mathbf{Z}/b_0 \subset \mathbf{C}^\times$.

Note that $\mathbf{P}_n(b_0, \ldots, b_n)$ is naturally identified with $\mathbf{P}_n(rb_0, \ldots, rb_n)$. Furthermore, if $(r, b_i) = 1$, $\mathbf{P}_n(rb_0, \ldots, b_i, \ldots, rb_n)$ is identified with $\mathbf{P}_n(b_0, \ldots, b_n)$ via the map $[z_0, \ldots, z_n] \mapsto [z_0, \ldots, z_i, \ldots, z_n]$. Also, $L_n(b_0; rb_1, \ldots, rb_n) \cong L_n(b_0; b_1, \ldots, b_n)$, and $L_n(b_0; b_1, \ldots, b_n) \cong L_n(b_0; rb_1, \ldots, b_i, \ldots, rb_n)$ if $(r, b_i) = 1$ $(i = 1, \ldots, n)$. Thus, we may assume $\gcd(b_0, \ldots, \check{b}_i, \ldots, b_n) = 1$ for all i.

Since both weighted projective spaces and pseudo-lens spaces are rational homology manifolds, intersection homology of these spaces with rational coefficients is isomorphic to ordinary rational homology, which is the same as the homology of ordinary projective space or the ordinary sphere. Thus our focus goes to torsion phenomena in intersection homology, which are much more complicated than torsion phenomena of ordinary homology and not yet well understood.

We first discuss the intersection homology of pseudo-lens spaces, then we will determine the intersection homology of weighted projective spaces by using the results about pseudo-lens spaces. We also