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EISENSTEIN-SERIES ON REAL, COMPLEX, AND QUATERNIONIC HALF-SPACES

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The real, complex, and quaternionic half-spaces are introduced in certain analogy with the Siegel half-space. The modified symplectic group acts on the attached half-space in the usual way. At first properties of these half-spaces considered as symmetric spaces are derived. Then a fundamental domain with respect to the modified modular group, which consists of integral modified symplectic matrices, is constructed. The behavior of convergence of the corresponding Eisenstein-series is determined carefully. The Fourier-coefficients of the Eisenstein-series are calculated explicitly, whenever the degree is sufficiently small.

Introduction. The present paper deals with half-spaces, which are built in analogy with the Siegel half-space, and the corresponding non-analytic Eisenstein-series. The roots can be traced back to C. L. Siegel's paper "Die Modulgruppe in einer einfachen involutorischen Algebra" [**30**]. A special case of these investigations is considered and continued by the examination of the Riemannian geometry as well as the attached Eisenstein-series.

To be more precise, throughout this paper let $\mathbf{F}$ stand for $\mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$, where $\mathbf{H}$ is the skew-field of real Hamiltonian quaternions. Just as in [**16**] let $r = r(\mathbf{F}) = \dim_{\mathbf{R}} \mathbf{F}$ and denote the standard basis of $\mathbf{F}$ over $\mathbf{R}$ by $1 = e_1, \ldots, e_r$. Given $a = \sum_{j=1}^{r} a_j e_j \in \mathbf{F}$, $a_j \in \mathbf{R}$, put $\operatorname{Re}(a) := a_1$ and let $a \mapsto \bar{a} = 2\operatorname{Re}(a) - a$ denote the canonical conjugation in $\mathbf{F}$. Then $A^{(n)}$, resp. $A \in \operatorname{Mat}(n; \mathbf{F})$, means that A is an $n \times n$ matrix with entries in $\mathbf{F}$ and A' denotes the transpose of A. The letter I is reserved for the identity matrix and 0 for the zero matrix of appropriate size. $\operatorname{GL}(n; \mathbf{F})$ stands for the group of units in the ring $\operatorname{Mat}(n; \mathbf{F})$.

The half-space $\mathscr{H}(n; \mathbf{F})$ consists of all $Z \in \operatorname{Mat}(n; \mathbf{F})$ such that $Z + \overline{Z}'$ becomes a positive definite Hermitian matrix. Thus $i\mathscr{H}(n; \mathbf{C})$ equals the Hermitian half-space, which was investigated by H. Braun [**3**]. But the remaining cases are related, because $\mathscr{H}(n; \mathbf{H})$ can always be embedded into the Hermitian half-space of degree $2n$.

The attached modified symplectic group $\operatorname{MSp}(n; \mathbf{F})$ consists of the automorphs of the symmetric matrix $Q = \left(\begin{smallmatrix} 0 & I \\ I & 0 \end{smallmatrix}\right)$, $I = I^{(n)}$, having the