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EMBEDDING 2-COMPLEXES IN $\mathbf{R}^4$

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Using Freedman's results it is not very hard to see that every finite acyclic 2-complex embeds in $\mathbf{R}^4$ tamely. In the present paper a relative version of this fact is proved. We also study when a finite acyclic 2-complex with one extra 2-cell attached along its boundary can be tamely embedded in $\mathbf{R}^4$.

Introduction. In 1955 A. Shapiro found a necessary and sufficient condition for the existence of embeddings of finite n-complexes in $\mathbf{R}^{2n}$ if $n > 2$ (see [**14**]) by defining an obstruction using the ideas of H. Whitney ([**15**]). The obstruction is not homotopy invariant and is in general quite hard to compute. It is well-known that any finite acyclic n-complex embeds in $\mathbf{R}^{2n}$ if $n \neq 2$ (see for example [**8**]). Not long ago it was proved in [**16**] that any finite n-complex K with $H^n(K)$ cyclic embeds in $\mathbf{R}^{2n}$ if $n > 2$.

It is known that any finite acyclic 2-complex can be embedded in $\mathbf{R}^4$ (see [**9**], compare also with [**11**]). In the present paper the following relative version is proved.

THEOREM 1. *Let K be a finite 2-complex obtained from a 2-complex L by adjoining one 2-cell e along its boundary. If $H^2(K) = 0$ then any π_1-negligible tame embedding of L into $\mathbf{R}^4$ can be extended to a π_1-negligible tame embedding of K into $\mathbf{R}^4$.*

REMARK. This result is the best possible in the following sense: there exists a π_1-negligible embedding of a finite acyclic 2-complex into $\mathbf{R}^4$ which cannot be extended over an additional 2-cell (see §3).

In §2 the following is proved:

THEOREM 2. *Let L be a finite acyclic 2-complex. Suppose K is obtained from L by attaching one additional 2-cell e_0 along its boundary. If a regular neighborhood of some complex $\tilde{K}$ which carries the second homology of K can be embedded in some orientable 3-manifold then K can be tamely embedded in $\mathbf{R}^4$.*

Note. $\tilde{K} \subset K$ carries the second homology of K if the inclusion $\tilde{K} \subset K$ induces an isomorphism $H_2(\tilde{K}) \approx H_2(K)$. A regular neighborhood