PACIFIC JOURNAL OF MATHEMATICS
Vol. 133, No. 2, 1988

DEHN-SURGERY ALONG A TORUS T^2-KNOT

ZYUN'ITI IWASE

A T^2-knot means a 2-torus embedded in a 4-manifold. We define *torus T^2-knots* in the 4-sphere S^4 as a generalization of torus knots in S^3. We classify them up to equivalence and study the manifolds obtained by Dehn-surgery along them.

1. Introduction. Dehn-surgery, which was introduced by Dehn [**1**], plays an important role in knot theory and 3-dimensional manifold theory. The classical Dehn-surgery is the operation of cutting off the tubular neighborhood $N = S^1 \times D^2$ of a knot in S^3 and of pasting it back via an element of $\pi_0 \operatorname{Diff} \partial N$, which is isomorphic to $\mathrm{GL}_2\mathbf{Z}$. Gluck-surgery [**2**] along a 2-knot in S^4 is a 4-dimensional version of Dehn-surgery. In this version, $N = S^2 \times D^2$ and $\pi_0 \operatorname{Diff} \partial N = (\mathbf{Z}/2)^3$. $(\mathbf{Z}/2)^2$ corresponds to the orientation reversing diffeomorphisms of S^2 and ∂D^2. Therefore Gluck-surgery yields at most one new manifold from one 2-knot and it is a homotopy 4-sphere (see [**2**]). Another 4-dimensional version is Dehn-surgery along a 2-torus embedded in S^4 [**7**], which we call a *T^2-knot* in this paper. In this version, $N = T^2 \times D^2$ and $\pi_0 \operatorname{Diff} \partial N = \mathrm{GL}_3\mathbf{Z}$. Countably many manifolds are obtained from one T^2-knot. A manifold obtained by Gluck-surgery is also obtained by Dehn-surgery along a T^2-knot (see Proposition 3.5). Dehn-surgery along an unknot is studied in [**7**], [**9**]. See also [**3**].

In this paper, we define a *torus T^2-knot* which is analogous to the torus knots in the classical knot theory, and classify them up to equivalence. Then we study the manifolds obtained by Dehn-surgeries along them.

Dehn-surgery along a torus knot is studied by Moser [**8**].

THEOREM 1.1. (*Moser* [**8**], *Propositions* 3.1, 3.2, 4.) *Assume that a Dehn-surgery of type* (α, β) *is performed along* $k(p, q)$, *the torus knot of type* (p, q). *Put* $|\sigma| = |pq\beta - \alpha|$. *The manifold obtained is denoted by* M.

(i) *If* $|\sigma| \neq 0$, *then* M *is a Seifert manifold with fibers of multiplicities* p, q, $|\sigma|$.

(ii) *If* $|\sigma| = 1$, *then* M *is a lens space* $L(|\alpha|, \beta q^2)$.