PACIFIC JOURNAL OF MATHEMATICS
Vol. 133, No. 2, 1988

PSEUDOCONVEX DOMAINS WITH PEAK FUNCTIONS AT EACH POINT OF THE BOUNDARY

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Under certain conditions, each point of the boundary of a smoothly bounded weakly pseudoconvex domain D in $\mathbf{C}^n$ is a peak point of $A^\infty(D)$.

1. Introduction. Let D be a bounded pseudoconvex domain with C^∞ boundary. We denote by $A^\infty(D)$ the set of holomorphic functions in D which have a C^∞ extension to $\overline{D}$. A compact subset E of ∂D is a peak set for $A^\infty(D)$ if there exists $f \in A^\infty(D)$ such that $f = 0$ on E and $\operatorname{Re} f > 0$ on $\overline{D} \backslash E$. Such a function will be called a strong support function for E. If $E = \{p\}$, p is a peak point for $A^\infty(D)$.

In **[6]**, **[18]** it is proved that each point of a strictly pseudoconvex domain is a peak point for $A^\infty(D)$ with a strong support function holomorphic in the neighborhood of $\overline{D}$ and in [7], **[17]** it is proved that each strongly pseudoconvex point of a weakly pseudoconvex domain with C^∞ boundary is a peak point for $A^\infty(D)$. These results fail in the case of weakly pseudoconvex domains **[4]**, **[13]**. Other results about smoothly varying peaking functions in pseudoconvex domains may be found in **[1]**, **[5]**, **[14]**.

If D is strictly pseudoconvex, Chaumat and Chollet proved in [3] that each closed subset of a peak set for $A^\infty(D)$ is a peak set for $A^\infty(D)$. The assertion is also true for bounded pseudoconvex domains in $\mathbf{C}^2$ of finite type **[15]** and for bounded pseudoconvex domains in $\mathbf{C}^2$ with isolated degeneracies **[11]** or with (NP) property **[12]**.

In **[16]** is given an example of convex domain in $\mathbf{C}^2$ not of finite type whose weakly pseudoconvex boundary points form a line segment which is a peak set for $A^\infty(D)$, but there is a point which is not a peak point for $A^\infty(D)$.

Here we prove that, under certain assumptions, each point of the boundary of a weakly pseudoconvex domain is a peak point for $A^\infty(D)$.

Some results of this paper were announced at the International Workshop Geometric and Quantitative Complex Analysis, Wuppertal 1986.