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UNIQUENESS FOR CERTAIN SURFACES OF PRESCRIBED MEAN CURVATURE

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The classical problem of liquid in a capillary tube concerns finding the minimum of a certain energy function, which leads to a surface of prescribed mean curvature. This paper weakens the hypotheses by considering local minima of the functional. It is shown that no new surfaces result.

1. Introduction. Although the theorems in this paper have fairly general hypotheses, they are motivated by the theory of capillary surfaces. The classical capillary problem is to study the shape of the surface formed by capillary action when a tube of general cross-section $\Omega \subseteq \mathbf{R}^n$ is placed into an infinite reservoir. The assumption that has customarily been made is that the surface is energy minimizing over compact perturbations. It is natural to ask whether a surface can exist which is a local minimum for the functional but not a global minimum. In other words, is it possible, in the classical problem, for an "exotic" capillary surface to exist which is stable with respect to small compact perturbations, but unstable with respect to a large compact perturbation? The paper proves that under fairly weak assumptions, the answer is "no".

Certainly there are other problems in capillarity with non-trivial local minima. The energy functional for the pendent drop (e.g. Wente [**10**]) has no minimum if arbitrarily large perturbations are allowed, since by falling to $-\infty$ a drop looses an infinite amount of potential energy. Another such situation is that of a drop trapped between two planes (Vogel [**8**]). For some values of the separation, the (stable) connected drop bridging the planes has a larger energy than a spherical cap on one plane.

The reason the assumption that the surface is energy minimizing over all perturbations has been made in the classical problem is that a result of Miranda [**5**] then implies that the surface is a graph. The idea of the proof is that replacing the set U occupied by the liquid by the set $U^s = \{(x,t) \colon t \leq \int_0^\infty \phi_U(x,\tau)\,d\tau\}$ will reduce perimeter and hence the relevant energy functional. Here $t = 0$ is chosen to lie below the