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HOLOMORPHICALLY CONVEX COMPACT SETS AND COHOMOLOGY

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Conditions are given, on a domain D of a Stein manifold X, for the cohomology groups $H^q(D;\mathscr{F})$ to be Fréchet-Schwartz spaces for every $q \geq 0$ and every coherent sheaf $\mathscr{F}$ on X.

Introduction. Let V be a complex space and $\mathscr{F}$ a coherent sheaf on V. It is well-known that we can endow $H^q(V;\mathscr{F})$ of a structure of topological vector space such that its separated $H^q(V;\mathscr{F})/\overline{0})$ is a Fréchet-Schwartz (F.S.) space. It is of some interest to know when $H^q(V;\mathscr{F})$ is itself F.S. (For instance it is possible, if the answer is affirmative, to prove a Künneth formula.) This is the case when V is Stein and $\mathscr{F}$ is any coherent sheaf on V, or when $V = X - K$, where X is Stein, K is a compact set with a fundamental system of Stein neighborhoods and $\mathscr{F}$ is a coherent sheaf on X. This is proved in ([**3**], Théorème 2.19, page 40).

In this paper we find conditions on a domain D, in a connected Stein manifold X of dimension $n > 1$, which are sufficient for the groups $H^q(D;\mathscr{F})$ to be F.S. and the cohomology groups with compact support $H^q_K(D;\mathscr{F})$ to be D.F.S. (Dual of Fréchet-Schwartz) for every $q \geq 0$ and every coherent sheaf $\mathscr{F}$ on X. These conditions turn out to be also necessary if the complex dimension of X is 2. Also we obtain a cohomology duality theorem for such domains.

Preliminaries. Consider a domain D in a connected Stein manifold X of dimension $n > 1$, let S be the union of the connected compact components of $X - D$ and $D' = D \cup S$; the set D' is open and connected ([**11**] page 30). Let K be a compact subset of X and $\mathscr{O}(K)$ be the direct limit $\varinjlim_{U \supseteq K} \mathscr{O}(U)$ with the inductive limit topology; let spec$\mathscr{O}(K)$ be the spectrum of $\mathscr{O}(K)$, i.e. the set of all nonzero continuous homomorphisms of the algebra $\mathscr{O}(K)$ into $\mathbf{C}$.

Following [**13**] we say that K is holomorphically convex if the usual evaluation map $g\colon K \to \text{spec}\mathscr{O}(K)$ given by $g(x)(f) = f(x)$ is bijective.