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A BONNESEN-STYLE INRADIUS INEQUALITY IN 3-SPACE

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A Bonnesen-style inradius inequality for convex bodies in E^3 is obtained using the method of inner parallel bodies. The inequality involves the volume, surface area and mean-width of the body.

I. Introduction. By a convex body we mean a compact convex set with non-empty interior. Let K be a planar convex body with area A, perimeter L, inradius r, and circumradius R. An inequality of Bonnesen states:

$$L^2 - 4\pi A \geq \pi^2(R-r)^2.$$

This inequality follows from

$$(1) \qquad 0 \geq A - xL + x^2\pi, \qquad r \leq x \leq R.$$

Equality holds in (1), at $x = r$, for the "sausage" bodies, that is, those bodies which are the Minkowski sum of a line segment and a ball (with radius r). At $x = R$ equality only holds for balls. For proofs of these inequalities see Eggleston [**5**, pp. 108–110].

An extension of Bonnesen's inradius inequality in the plane to higher dimensions began with the conjecture by Wills [**11**] that

$$0 \geq V - rS + (n-1)r^n\omega_n.$$

In this paragraph, V will represent the n-dimensional volume of a convex body in E^n, S its n-dimensional surface area, and ω_n the volume of the unit n-ball. The conjecture was proved simultaneously by Bokowski [**1**] and Diskant [**4**]. Equality holds only for the n-balls. Osserman [**8**] showed that

$$(2) \qquad 0 \geq V - rS + (n-1)r^2 \sqrt[n-1]{\omega_n(S/n)^{n-2}}$$

where equality also holds only for the n-balls. This inequality is the sharper because a translate of rB is contained in K.

The results of this paper will be limited to the case $n = 3$. The volume and surface area of the convex body K will be represented by $V(K)$ and $S(K)$. The unit 3-ball centered at the origin is denoted B and $V(B) = \omega$. The functional $M(K)$ will be proportional to the