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HOMOGENEOUS SPECTRAL SETS AND LOCAL-GLOBAL METHODS IN BANACH ALGEBRAS

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The search for topological characterization of algebraic properties of commutative Banach algebras has often led to the calculation of cohomology groups of its spectrum.

In the present paper, a tool is developed which permits the application of sheaf cohomological methods to such problems by lifting sequences of complex analytic manifolds to sequences of spectral sets and cohomology groups.

Introduction. Suppose A is a commutative, unital Banach algebra, with spectrum $X(A)$, and let

$$\theta : \mathscr{O}(X(A)) \to A$$

be the global functional calculus defined by Craw [**3**]. Also, let M denote a complex analytic submanifold of $\mathbf{C}^n$.

While searching for characterizations of Čech cohomology groups of $X(A)$, Novodvorskii [**8**] presented the spectral sets

$$A^M = \{a = (a_1, \dots, a_n) \in A^n : \operatorname{sp}(a_1, \dots, a_n) \subset M\}.$$

Later Taylor used in [**10**]

$$A_M = \{a = (a_1, \dots, a_n) \in A^n : a = \theta^n(f), \text{ with } f \in \mathscr{O}(X(A), M)\}.$$

As it turns out, such sets have a very rich structure. It was proved in [**6**] that they are Banach manifolds modelled on the projective A-modules of rank $= \dim(M)$.

Our aim in this note is to extend the definition of spectral set to include manifolds such as the complex projective line, or complex Grassmann manifolds, which are not submanifolds of $\mathbf{C}^n$. We will obtain, for example, a good definition of meromorphic elements of a Banach algebra. By "good" we mean that our definition will be intrinsic, will coincide with Taylor's definition for submanifolds of $\mathbf{C}^n$, and will verify a Novodvorskii-Taylor type theorem [**10**] when the manifold is a homogeneous space. We shall also generalize Craw's global function calculus θ to a map

$$\theta_M : \mathscr{O}(X(A), M) \to A_M.$$