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INVARIANT SUBSPACES OF $\mathscr{H}^2$ OF AN ANNULUS

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Fully invariant subspaces of the Hardy class $\mathscr{H}^2(\mathrm{G})$ on a multiply connected domain $\mathrm{G} \subset \mathbf{C}$, are those $\mathscr{M}$ such that

$$f(x) \in \mathscr{M} \Rightarrow Q(z)f(z) \in \mathscr{M},$$

for all rational functions Q whose poles are in the complement of $\overline{\mathrm{G}}$. Simply invariant subspaces are those $\mathscr{M}$ such that

$$f(z) \in \mathscr{M} \Rightarrow zf(z) \in \mathscr{M}.$$

Although the structure of the fully invariant subspaces is well known as a result of the work of Sarason, Hasumi, and Voichick, little work has been done on subspaces simply invariant but not fully invariant. In this paper we consider the special case $\mathrm{G} = \mathrm{A}$, where A denotes the annulus $\{z \in \mathbf{C}: 1 < |z| < R\}$. We classify the simply invariant (closed) subspaces $\mathscr{M}$ of $\mathscr{H}^2(\mathrm{A})$.

0. Introduction and statement of results. The fully invariant subspaces of the Hardy class $\mathscr{H}^2(\mathbf{G})$ on a multiply connected domain $\mathbf{G} \subset \mathbf{C}$, as well as some of the simply invariant ones, have been classified (cf., [12], [23], [25], and [27]). Fully invariant subspaces are those $\mathscr{M}$ such that

$$f(z) \in \mathscr{M} \Rightarrow Q(z)f(z) \in \mathscr{M},$$

for all rational functions Q whose poles are in the complement of $\overline{\mathbf{G}}$. Simply invariant subspaces are those $\mathscr{M}$ such that

$$f(z) \in \mathscr{M} \Rightarrow zf(z) \in \mathscr{M}.$$

In this paper we consider the special case $\mathbf{G} = \mathbf{A}$, where $\mathbf{A}$ denotes the annulus $\{z \in \mathbf{C}: 1 < |z| < R\}$. We extend the results of Royden [23] by classifying the simply invariant subspaces $\mathscr{M}$ of $\mathscr{H}^2(\mathbf{A})$. Here and throughout this paper "subspace" means "closed subspace". If we also have $z^{-1}f(z) \in \mathscr{M}$ for all $f \in \mathscr{M}$, we say that $\mathscr{M}$ is doubly invariant or fully invariant. Note that this use of "fully invariant" is consistent with the use above.

Saragon [25], Hasumi [12], and Voichick [27, 28] were the original investiagators of fully invariant subspaces of $\mathscr{H}^2(\mathbf{A})$. They characterized them, as well as the subspaces of $\mathscr{L}^2(\partial\mathbf{A})$ which are invariant