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THREEFOLDS WHOSE HYPERPLANE SECTIONS ARE ELLIPTIC SURFACES

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In this paper we classify pairs (X, S) where X is a smooth complex projective threefold and S is a smooth ample divisor in X. Moreover S is elliptic and $\kappa(S) = 1$. We use the logarithmic Kodaira dimension of (X, S) as the basis of classification. Sommese studied such pairs in "The birational theory of hyperplane sections of projective threefolds" where he showed that such pairs (X, S) can be reduced to (X', S'), where S' is ample in X', and S' is minimal model of S. In the case when S is elliptic, with $h^{1,0}(S) \neq 0$ he showed that one obtains a surjective morphism p, from X onto a smooth curve Y such that this morphism restricted to S is a reduced elliptic fibration. Shepherd-Barron proved the same result using Mori's methods without the restriction on $h^{1,0}(S)$. We state these results in §0.

We show that the general fibres of p are del Pezzo surfaces and classify these in the case where they are of degrees 1, 2, 3, 4, 7, 8 and 9. We show that in the degree 9 case that it is indeed a $\mathbf{P}^2$-bundle over Y. In the degree 8 ($\cong \mathbf{P}^1 \times \mathbf{P}^1$) case we have a birational morphism to a $\mathbf{P}^2$-bundle.

0. Notation and background material. For the most part we will follow the notation in [**Ha 1**] as closely as possible. For the convenience of the reader we have included as much as possible. In this section we state most of the theorems that have been used.

(0.1) If $\mathcal{S}$ is a sheaf of abelian groups on a topological space X, then the global sections of $\mathcal{S}$ over X are denoted by $\Gamma(\mathcal{S})$, or by $\Gamma(X, \mathcal{S})$ in case of ambiguity.

(0.2) All manifolds and spaces are complex analytic, unless specified otherwise. All dimensions are over $\mathbf{C}$. Complex analytic is often abbreviated to analytic. The sheaf of X is denoted by $\mathcal{O}_X$. We do not distinguish between a holomorphic vector bundle on X, and its sheaf of holomorphic sections. Hence a tensor product of a vector bundle and a coherent analytic sheaf is actually the appropriate sheaves being tensored together over $\mathcal{O}_X$.

(0.3) If $\mathcal{S}$ denotes a coherent analytic sheaf on X, then $\chi(X, \mathcal{S})$ or $\chi(\mathcal{S})$ denotes its Euler characteristic.

(0.4) When the exact dimension of a projective space is irrelevant, then we denote it by $\mathbf{P}$.