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THE ADAMS SPECTRAL SEQUENCE OF THE REAL PROJECTIVE SPACES

RALPH L. COHEN, WEN-HSIUNG LIN AND MARK E. MAHOWALD

In this paper we study the mod 2 **Adams spectral sequence for the infinite real projective space** $P = \mathbf{R}P^\infty$.

We recall ([1]) that the spectral sequence starts with

$$E_2^{s,t} = \mathrm{Ext}_A^{s,t}(\tilde{H}^*(P), \mathbf{Z}/2)$$

and converges to the stable homotopy ${}_2\pi_*^s(P) = \pi_*^s(P)$ where A denotes the mod 2 Steenrod algebra and $\tilde{H}^*(P)$ is the reduced mod 2 cohomology of P. We simply write $\mathrm{Ext}_A^{s,t}(P)$ for $\mathrm{Ext}_A^{s,t}(\tilde{H}^*(P), \mathbf{Z}/2)$ and occasionally we abbreviate by ASS for "Adams spectral sequence".

Roughly, our main results consist of (1) a complete description of $\mathrm{Ext}_A^{s,*}(P)$ for $0 \le s \le 2$ and also for $s = 3$ modulo indecomposable elements, (2) the determination of which classes in a substantial portion of $\mathrm{Ext}_A^{1,*}(P)$ can detect homotopy elements in $\pi_*^s(P)$ (Adams's Hopf invariant Theorem solves the problem for $\mathrm{Ext}_A^{0,*}(P)$) and (3) the construction of some infinite families in $\pi_*^s(P)$ at low Adams filtrations analogous to the ones in the 2-adic stable homotopy of spheres ${}_2\pi_*^s$ constructed in [9], [12] and [18], [22].

These Ext calculations were necessary in the work on the Kervaire invariant in [12]. The results are not surprising, but proving them is surprisingly difficult. In particular we make use of a calculational method that may be of independent interest.

To precisely state the results we first recall that the cohomology $\mathrm{Ext}_A^{*,*} = \mathrm{Ext}_A^{*,*}(\mathbf{Z}/2, \mathbf{Z}/2)$ of the Steenrod algebra A is a commutative bigraded algebra over $\mathbf{Z}/2$ and that $\mathrm{Ext}_A^{*,*}$ for $0 \le s \le 3$ is generated by $h_i \in \mathrm{Ext}_A^{1,2^i}$ $(i \ge 0)$ and $c_i \in \mathrm{Ext}_A^{3,2^{i+3}+2^{i+1}+2^i}$ with relations $h_i h_{i+1} = 0, h_{i+1}^3 = h_i^2 h_{i+2}$ and $h_i h_{i+2}^2 = 0$ where h_i corresponds to the Steenrod square $\mathrm{Sq}^{2^i} \in A$. The mod 2 cohomology $H^*(P)$ is a polynomial algebra $\mathbf{Z}/2[x]$ in one variable x with $\deg x = 1$ on which A acts by $\mathrm{Sq}^k x^i = \binom{i}{k} x^{i+k}$. One easily proves that $\{x^{2^i-1} \mid i \ge 1\}$ is a minimal set of generators of $\tilde{H}^*(P)$ over A. The non-zero class in $\mathrm{Ext}_A^{0,2^i-1}(P) = \mathbf{Z}/2$ corresponding to x^{2^i-1} is denoted by $\hat{h}_i$. The first part of the