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ANNULAR BUNDLES

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This paper is devoted to the study of a particular kind of complex manifold with non-trivial topology: holomorphic fiber bundles with fibers biholomorphic to plane annuli.

0. Introduction. In recent years, some work has been done on function theory in complex manifolds with non-trivial topology. Two different approaches have been developed, a variational one and a purely complex-theoretical one.

The origins of the former (due essentially to Bedford and Burns; cf. **[BBu]** and **[B1]**) lie in the work of Landau and Osserman **[LO1, LO2]** on multiply connected Riemann surfaces. They defined an invariant norm on the homology group of the surface, using a particular family of harmonic functions. The solution of an associated extremal problem is a *harmonic measure* of the surface. The invariance properties of this function can be used to get several results in function theory, for instance the classification of the plane annuli.

Bedford and Burns, in **[BBu]**, developed a similar theory in bounded domains of $\mathbf{C}^n$ of the form $D_1 \backslash D_2$, where D_1 and D_2 were smooth strongly pseudoconvex domains with $D_2 \subset\subset D_1$. They used an invariant norm on the homology groups defined by Chern et al. in **[CLN]**, and the solution of a particular complex Monge-Ampère equation as harmonic measure. Bedford, in **[B1]**, studied complex manifolds of (complex) dimension n with $H_n(X, \mathbf{R}) \neq (0)$ using several other invariant norms on $H_n(X, \mathbf{R})$.

The second approach (due essentially to Bedford, again, and Mok; cf. **[B2]** and **[Mo]**) is based on the classical theory of Stein manifolds, and is devoted to the study of Stein manifolds of (complex) dimension n with $H_n(X, \mathbf{R}) \neq (0)$. In particular, Mok proved that, under some mild assumptions, a holomorphic map of such a Stein manifold into itself inducing an isomorphism of $H_n(X, \mathbf{R})$ is an automorphism.

These methods do not work on manifolds with non-trivial homology only in low dimensions. For instance, we do not get any result on the simplest example of non-contractible strongly pseudoconvex domain,