PACIFIC JOURNAL OF MATHEMATICS
Vol. 134, No. 2, 1988

K-THEORY FOR GRADED BANACH ALGEBRAS II

A. VAN DAELE

Let A be a real or complex Banach algebra and assume that A is equipped with a continuous automorphism α such that α^2 is the identity. In "K-theory for graded Banach algebras I" we have associated a group $K(A)$ to such a pair (A, α). In this paper we prove that this group $K(A)$ is isomorphic with $K(SA\hat{\otimes}C)$ where SA is the algebra of continuous functions $f : [0,1] \to A$ with $f(0) = f(1) = 0$ and equipped with pointwise operations and where $SA\hat{\otimes}C$ denotes the graded tensor product of SA with the Clifford algebra $C = C^{0,1}$. The periodicity of Clifford algebras is used to show that $K(S^8A) = K(A)$ in general and $K(S^2A) = K(A)$ in the complex case. All this gives rise to an important periodic exact sequence associated to an algebra A and an invariant closed ideal I with

$$K(I) \to K(A) \to K(A/I) \to K(I\hat{\otimes}C) \to K(A\hat{\otimes}C) \to K(A/I\hat{\otimes}C)$$

as its typical part. The usual 6-term periodic exact sequence with K_0 and K_1 is a special case of this sequence.

1. Introduction. In a previous paper we have defined an abelian group $K(A)$ for any real or complex Banach algebra A equipped with a $\mathbf{Z}_2$-grading [4]. For convenience we work with an involutive automorphism α that determines the grading in the sense that $\deg a = 0$ if $\alpha(a) = a$ and $\deg a = 1$ if $\alpha(a) = -a$. The main result in [4] is the usual exact sequence

$$K(SI) \to K(SA) \to K(S(A/I)) \to K(I) \to K(A) \to K(A/I)$$

for any invariant closed two-sided ideal I of A. As usual SA is the algebra of continuous functions $f : [0,1] \to A$ such that $f(0) = f(1) = 0$ with pointwise operations and supremum norm.

In this paper we will obtain another important exact sequence. It is related to the following lifting problem. As above let I be an invariant closed two-sided ideal of A. Denote by π the quotient map and also use the symbol α for the induced involution on A/I. Assume that A has an identity and take an element $x \in A/I$ such that $x^2 = 1$ and $\alpha(x) = -x$. Of course there is an element $a \in A$ such that $\pi(a) = x$ and by taking $\frac{1}{2}(a - \alpha(a))$ we may even assume that also $\alpha(a) = -a$. In general however it will not be possible to find a lifting a such that also $a^2 = 1$.