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# PSEUDOCONVEX CLASSES OF FUNCTIONS I. PSEUDOCONCAVE AND PSEUDOCONVEX SETS

ZBIGNIEW SLODKOWSKI

**An axiomatic definition of a pseudoconvex class of functions is developed. The models include classes of subharmonic, plurisubharmonic, $q$-plurisubharmonic, convex and $q$-convex functions, and many others. Notions of dual class of functions and of pseudoconcave and pseudoconvex sets are introduced and studied. The results have applications to complex interpolation of normed spaces, given elsewhere.**

**Introduction.** It is well known that classes of subharmonic, plurisubharmonic, and convex functions share many properties including, in particular, existence and uniqueness of the solution to the generalized Dirichlet problem. Classes of $q$-plurisubharmonic functions, studied by Hunt and Murray [**4**] and the author [**8**], follow the same pattern. During his work on [**8**] the author realized that most of these similarities are consequences of few simple properties.

Namely, each of these classes is a sheaf on $R^N$, consisting of upper semicontinuous functions with local maximum property, preserved by multiplication by positive constants. The limit of a decreasing sequence of functions of a given class belongs to it, as does supremum of several functions of this class. While only some of these classes are closed with respect to addition, all of them satisfy the following, weaker, property.

(0.1) Whenever $u$ is a function of a given class, and $v$ is a convex function, then $u + v$ is a function of the same class.

All of these classes are also translation invariant, that is (0.2) whenever $u\colon U \to [-\infty, +\infty)$ is of given class, then $u_y$ is of the same class, for every $y \in R^N$, where $u_y(x) = u(x - y)$, $x \in U + y$.

We will tentatively call any class of functions satisfying the above conditions a *translation invariant pseudoconvex class on $R^N$*. (See Examples 2.1–2.3 below.)

It turns out that considerable parts of the theory of plurisubharmonic functions, polynomially convex sets, various types of pseudoconvex and pseudoconcave sets, and some parts of potential theory