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TOPOLOGICAL ENTROPY AND RECURRENCE OF COUNTABLE CHAINS

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We consider a symbolic dynamical system (X, σ) on a countable state space. We introduce a kind of topological entropy for such systems, denoted h^*, which coincides with usual topological entropy when X is compact. We use a pictorial approach, to classify a graph Γ (or a chain) as transient, null recurrent, or positive recurrent. We show that given $0 \le \alpha \le \beta \le \infty$, there is a chain whose h^* entropy is β and where Gurevic entropy is α. We compute the topological entropies of some classes of chains, including larger chains built up from smaller ones by a new operation which we call the Cartesian sum.

Introduction. The importance of subshifts of finite type in ergodic theory and dynamical systems is well known. One needs also to study chains on a countably infinite set in order to analyze problems in various fields such as differentiable dynamics, coding for magnetic recording, nonuniqueness of equilibrium states in statistical mechanics, formal languages and automata, or even to analyze arbitrary subshifts. (See [3], [9], [6], [2], [1], [10], respectively.)

Let Γ be a strongly connected directed graph on a countable set of vertices $S = \{s_1, s_2, \dots\}$, and let

$$X(\Gamma) = \{x \in S^z \mid \text{ for all } i, \text{ there is an edge in } \Gamma \text{ from } x_i \text{ to } x_{i+1}\}.$$

If S has the discrete topology and S^z the product topology, then in the induced topology $X(\Gamma)$ (or simply X), together with the shift transformation σ defined by $(\sigma x)_i = x_{i+1}$ for all i, is a (non-compact) dynamical system, called the *chain* determined by the directed graph Γ. The topological entropy of X may be determined using Bowen's definition, to obtain $h_B(X)$ (see [8] for a definition). This definition depends on the metric we put on X. We consider the following two metric spaces.

1. For $x, y \in X$ define

$$d_1(x, y) = \sum_i \frac{1 - \delta(x_i, y_i)}{2^{|i|}},$$

where $\delta(s, t) = 1$ if $s = t$ and 0 if $s \neq t$.