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ASYMPTOTIC EXPANSION AT A CORNER FOR THE CAPILLARY PROBLEM

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Consider the solution of capillary surface equation over domains with corners. It is shown that there exists an asymptotic expansion of the solution at the corner if the corner angle 2α satisfies $0 < 2\alpha < \pi$ and $\alpha + \gamma > \pi/2$ where $0 < \gamma \leq \pi/2$ is the contact angle between the surface and the container wall. It is assumed that the corner is bounded by lines. The leading terms of the expansion are calculated and properties of the remainder are given.

1. Introduction and results. We consider the non-parametric capillary problem in the presence of gravity. One seeks a surface $S\colon u = u(x)$, defined over a bounded base domain $\Omega \subset \mathbf{R}^2$, such that S meets vertical cylinder walls over the boundary $\partial\Omega$ in a prescribed constant angle γ. This problem leads to the equations, see Finn [2],

$$\operatorname{div} Tu = \kappa u \quad \text{in } \Omega \tag{1.1}$$

$$\nu \cdot Tu = \cos\gamma \text{ on the smooth parts of } \partial\Omega, \tag{1.2}$$

where

$$Tu = \frac{Du}{\sqrt{1+|Du|^2}},$$

$\kappa = \text{const.} > 0$ and ν is the exterior unit normal on $\partial\Omega$.

Let the origin $x = 0$ be a corner of Ω with the interior angle 2α satisfying

$$0 < 2\alpha < \pi. \tag{1.3}$$

For simplicity, let us assume that the corner is bounded by lines near $x = 0$, see Figure 1.

We choose $0 < R_0 < 1$ small enough such that $\Omega_{R_0} = \Omega \cap B_{R_0}$ with $B_R = \{x \in \mathbf{R}^2 / x_1^2 + x_2^2 < R\}$ is a circular sector. Furthermore, we assume that

$$0 < \gamma < \pi/2 \tag{1.4}$$

and

$$\alpha + \gamma > \pi/2. \tag{1.5}$$