PACIFIC JOURNAL OF MATHEMATICS
Vol. 134, No. 2, 1988

POINT SEPARATION BY BOUNDED ANALYTIC FUNCTIONS OF A COVERING RIEMANN SURFACE

MIKIHIRO HAYASHI AND MITSURU NAKAI

Results of both positive and negative directions on the point separation by bounded analytic functions of two-sheeted covering Riemann surfaces are given when the points of base Riemann surfaces are separated by bounded analytic functions.

1. Introduction. Let R be a Riemann surface and $H^\infty(R)$ the set of bounded analytic functions on R. In the study of bounded analytic functions on Riemann surfaces, one of the basic problems is to determine when the points of R are separated by $H^\infty(R)$. Here we say that $H^\infty(R)$ separates the points of R (or $H^\infty(R)$ is *separating*) if for any pair of distinct points a and b of R there exists an f in $H^\infty(R)$ with $f(a) \neq f(b)$. Although we do not have any satisfactory answer to the problem, there is a very general result on the point separation by an algebra of analytic functions by Royden [5]. Applied to the present case the Royden result amounts to saying that if a Riemann surface $\tilde{R}$ admits a nonconstant bounded analytic function, then there is a quotient Riemann sueface R of $\tilde{R}$ with a quotient map ψ of $\tilde{R}$ onto R such that $H^\infty(R)$ is isomorphic to $H^\infty(\tilde{R})$ via the correspondence $f \to f \circ \psi$ and such that $H^\infty(R)$ is *weakly separating*, by which we mean that $H^\infty(R)$ separates the points of R except for a countable subset of R. At present, the gap between this general result of Royden and our knowledge on concrete examples is wide. For this reason it might be natural to consider the problem in the following special case as an experimental study.

Suppose there is given a Riemann surface R such that $H^\infty(R)$ is separating. For a Riemann surface $\tilde{R}$ with a holomorphic proper mapping ψ of $\tilde{R}$ onto R (i.e., $\tilde{R}$ is a ramified finitely sheeted unlimited covering surface of R), we ask when $H^\infty(\tilde{R})$ is separating.

The problem has been considered by Selberg [8], and later by Stanton [9] when the base domain R is the open unit disk, and then by Segawa [7] when R is a Riemann surface of Parreau-Widom type. In this note, we continue to study the problem in the case when the base