PACIFIC JOURNAL OF MATHEMATICS
Vol. 134, No. 2, 1988

AN INDEX CHARACTERIZATION OF THE CATENOID AND INDEX BOUNDS FOR MINIMAL SURFACES IN $\mathbf{R}^4$

SHIU-YUEN CHENG AND JOHAN TYSK

The index of a minimal surface is defined to be the number of negative eigenvalues of the operator corresponding to second variation of area. In the present paper, we characterize the catenoid as the only complete oriented minimal surface in $\mathbf{R}^3$ of index one with embedded ends. We also obtain upper bounds for the index of minimal surfaces in $\mathbf{R}^4$, in terms of the total curvature of the surface.

1. Introduction. The index of a minimal surface measures how far the surface is from being stable, with index equal to zero corresponding to stability. For noncompact surfaces M, one defines the index as the limit of the indices of an exhausting sequence of compact domains D in M. The index of a domain D is the number of negative eigenvalues of the Dirichlet eigenvalue problem on D for the operator corresponding to second variation of area. For oriented minimal surfaces in $\mathbf{R}^3$, this eigenvalue problem has the form

$$(\Delta - 2K)\phi + \lambda\phi = 0 \quad \text{on } D, \qquad \phi|_{\partial D} = 0,$$

where K is the Gaussian curvature of M, and Δ the Laplace-Beltrami operator on M.

D. Fischer-Colbrie and R. Schoen [**FC-S**], showed that the only complete oriented minimal surface in $\mathbf{R}^3$ of index zero is a plane. Fischer-Colbrie [**FC**], then showed that the index of a complete oriented minimal surface in $\mathbf{R}^3$ is finite if and only if the total curvature of the surface is finite. Her results, however, give no explicit information on how the index varies with the total curvature. In [**T1**], it is shown, using heat-kernel techniques, that the index is bounded by an explicit constant times the total curvature.

In §2 of this paper, we obtain a weaker type of bound from below for the index of minimal surfaces in $\mathbf{R}^3$. We show that all oriented complete minimal surfaces with embedded ends, except the plane and the catenoid, have index at least two. This yields as a corollary the characterization of the catenoid as the only complete oriented minimal surface of index one, with embedded ends.