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EXTENSION OF FLOWS VIA DISCONTINUOUS FUNCTIONS

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We consider flows (X, T) with X compact Hausdorff, and suitable discontinuous functions $f : X \to W$ where W is an arbitrary compact Hausdorff space. A ring extension of the ring of all continuous complex valued functions on $X(C(X))$ is formed and equipped with a norm. The Gelfand-Naimark theorem is applied to the completion of this normed ring to produce an almost one-to-one extension $\rho : (X_f, T) \to (X, T)$.

The question of isomorphism of flows (X_f, T) and (X_g, T) corresponding to functions f and g is discussed, as well as the lifting of dynamical properties from (X, T) to (X_f, T). Extension of flows via classes of discontinuous functions is considered, showing that no new examples arise in this way. A characterization theorem for extensions is proved when T is locally compact Hausdorff, showing that every minimal almost one-to-one extension of (X, T) can be obtained using our construction.

Introduction. In this paper we are concerned with creating extensions of flows by means of discontinuous functions. In essence the device is to add a suitable discontinuous function to a ring of continuous functions, obtaining a new structure space, in such a way that a new flow is generated which is an extension of the original one. Markley investigated extensions involving splitting along a single orbit by a somewhat different approach in [6].

The motivation for this theory is two fold. One hopes to modify existing examples to introduce new desired properties. We are able to introduce any compact metric space as a fibre in the extension in a similar way to that used by N. G. Markley in his situation. The classical Sturmian discrete flows involving adding two point fibres to a minimal circle rotation, thus obtaining highly proximal flows from equicontinuous ones, are probably the best known examples of this type of extension.

Another use for this extension process is to build models of the original flow. The initial flow is replaced by one of a desired type which is still close to the original in the sense that appropriate dynamical properties lift and the lifting map is "almost" an isomorphism i.e. it