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IRREDUCIBILITY OF UNITARY PRINCIPAL SERIES FOR COVERING GROUPS OF SL$(2, k)$

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This paper establishes the irreducibility of certain unitary principal series representations of covering groups of SL$(2, k)$, where k is a p-adic field, with p odd.

0.1. The theory of automorphic forms on covering groups of reductive groups over number fields has been shown to have important arithmetical applications [5], [3]. It is thus natural to study the representation theory of covering groups over p-adic fields. The representation-theoretic results which seem to be most applicable to automorphic forms are those concerning the reducibility of non-unitary principal series. The main results concern GL(n) and have been established by Kazhdan and Patterson [3]. In this paper we undertake the study of the unitary principal series by establishing complete reducibility results for n-sheeted covering groups of SL$(2, k)$, where k is a p-adic field containing the nth roots of unity. For ease of exposition, we assume p is odd. The proof uses a detailed analysis in the Fourier transform realization. This procedure is well known, but carrying out the details in the general case is rather involved. In particular, a careful study of matrix-valued Bessel functions is necessary.

The main result of the paper states that when n is even, all unitary principal series are irreducible, and that when n is odd, the only reducible ones are those induced from non-trivial characters of order 2 of k^{x}. The reducibility results in the case of n odd follow from [6]; the proofs here deal with the irreducibility. These results can easily be applied to establish the reducibility of certain unitary principal series of covering groups of p-adic Chevalley groups. A more complete study, however, requires a completeness theorem like that proved by Harish-Chandra for reductive p-adic groups.

1.1. Let k be a p-adic field. Let n be a positive integer and assume k contains the nth roots of unity. Let $(\, , \,)$ be the norm residue symbol of degree n. Let $G = \mathrm{SL}(2, k)$. There is a covering group $\tilde{G}$ defined as