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BARYCENTRIC SUBDIVISIONS

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A characterization is given of simplicial polytopes that are barycentric subdivisions of regular CW spheres. It is shown that barycentric subdivisions of connected polyhedral complexes with at least two facets determine the underlying complex uniquely up to duality. Connections with the algorithmic theory of comparability graphs are discussed. The f-vectors of regular CW spheres are characterized.

1. Introduction. Polytopes arise in many areas of mathematics, both pure and applied. Frequently the use of polytopes depends on an understanding of their combinatorial, as opposed to metrical, properties. Much progress has been made in the combinatorial study of polytopes in recent years. The best understood class of polytopes is that of simplicial polytopes. Attempting to use simplicial polytopes to study arbitrary polytopes leads to the study of barycentric subdivisions. This paper studies barycentric subdivisions of geometric cell complexes with a focus on polytopes.

We consider all structures to be in Euclidean space; the definitions are accordingly narrow.

DEFINITION 1 **[6]**. An *incidence polytope* is a graded partially ordered set with $\hat{0}$ and $\hat{1}$ satisfying the following properties:

(1) If C and D are maximal chains, then there is a finite sequence of maximal chains, $C = C_0, C_1, \dots, C_k = D$, where for each i $(0 \leq i \leq k)$ C_i contains $C \cap D$, and C_i and C_{i+1} differ in exactly one element.

(2) If x and z are elements of the poset with $x < z$ and $\operatorname{rank}(z) - \operatorname{rank}(x) = 2$, then there are exactly two elements y such that $x < y < z$.

An incidence polytope is said to have dimension n if the maximal chains contain $n+2$ elements (including $\hat{0}$ and $\hat{1}$).

For the next definition we use the following terminology: a set in $\mathbf{R}^n$ is an *open cell* if it is homeomorphic to the interior of the k-dimensional unit ball for some k; an open cell is regular if its closure is homeomorphic to the closed unit ball.