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A SIMPLE FORMULA FOR CONDITIONAL WIENER INTEGRALS WITH APPLICATIONS

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Yeh's inversion formula for conditional Wiener integrals is very complicated to apply when the conditioning function is vector-valued. This paper gives a very simple formula for such integrals. In particular, we express the conditional Wiener integral directly in terms of an ordinary (i.e., nonconditional) Wiener integral. Using this new formula, it is very easy to generalize the Kac-Feynman formula and also to obtain a Cameron-Martin type translation theorem for general conditional Wiener integrals.

1. Introduction. Consider the Wiener measure space $(C[0,T], \mathscr{F}^*, m_w)$ where $C[0,T]$ is the space of all continuous functions x on $[0,T]$ vanishing at the origin. For each partition $\tau = \tau_n = \{t_1, \dots, t_n\}$ of $[0,T]$ with $0 = t_0 < t_1 < \cdots < t_n = T$, let $X_\tau: C[0,T] \to \mathbf{R}^n$ be defined by $X_\tau(x) = (x(t_1), \dots, x(t_n))$. Let $\mathscr{B}^n$ be the σ-algebra of Borel sets in $\mathbf{R}^n$. Then a set of the type

$$I = \{x \in C[0,T]: X_\tau(x) \in B\} \equiv X_\tau^{-1}(B), \qquad B \in \mathscr{B}^n,$$

is called a Wiener interval (or a Borel cylinder). It is well known that

$$(1.1) \qquad m_w(I) = \int_B K(\tau, \vec{\xi})\, d\vec{\xi},$$

where

$$(1.2) \quad K(\tau,\vec{\xi}) = \left\{\prod_{j=1}^{n} 2\pi(t_j - t_{j-1})\right\}^{-1/2} \exp\left\{-\frac{1}{2}\sum_{j=1}^{n}\frac{(\xi_j - \xi_{j-1})^2}{t_j - t_{j-1}}\right\},$$
$$\vec{\xi} = (\xi_1, \dots, \xi_n), \quad \text{and} \quad \xi_0 = 0.$$

m_w is a probability measure defined on the algebra $\mathscr{F}$ of all Wiener intervals and m_w is extended to the Carathéodory extension $\mathscr{F}^*$ of $\mathscr{F}$. Let $\mathscr{F}_\tau$ be the σ-algebra generated by the set $\{X_\tau^{-1}(B): B \in \mathscr{B}^n\}$ with τ fixed. Then, by the definition of conditional expectation (see Tucker