PACIFIC JOURNAL OF MATHEMATICS
Vol. 135, No. 2, 1988

AMENABILITY AND KUNZE-STEIN PROPERTY FOR GROUPS ACTING ON A TREE

CLAUDIO NEBBIA

We characterize the amenable groups acting on a locally finite tree. In particular if the tree is homogeneous and the group G acts transitively on the vertices then we prove that G is amenable iff G fixes one point of the boundary of the tree. Moreover we prove that a group G which acts transitively on the vertices and on an open subset of the boundary is either amenable or a Kunze-Stein group.

1. Introduction and notations. Let X be a locally finite tree, that is, a connected graph without circuits such that every vertex belongs to a finite set of edges. Let V be the set of vertices and E the set of edges. If v_1 and v_2 are in V, let $[v_1, v_2]$ be the unique geodesic connecting v_1 to v_2; the distance $d(v_1, v_2)$ is defined as the length of the geodesic $[v_1, v_2]$. Let $\mathrm{Aut}(X)$ be the locally compact group of all isometries of X and, for $x \in V$, let K_x be the stability subgroup of x; K_x is a compact open subgroup of $\mathrm{Aut}(X)$. Let Ω be the boundary of the tree, that is, the set of equivalence classes of sequences of distinct vertices $\{s_n\}$, $n = 0, 1, 2, \ldots$, such that $[s_i, s_{i+1}]$ is an edge for every $i = 0, 1, 2, \ldots, n, \ldots$ (two such sequences are said to be equivalent if they have infinitely many common vertices). Ω is a compact metric space; every class of Ω is called an end of the tree. If $x_0 \in V$ and $\omega_0 \in \Omega$, there exists a unique geodesic $[x_0, \omega_0]$ from x_0 to ω_0, that is, a unique sequence $\{s_n\}$ of distinct vertices $\{s_0, s_1, \ldots, s_n, \ldots\}$ in the class ω_0 such that $s_0 = x_0$. Hence Ω can also be regarded, as the set of infinite sequences starting from any fixed vertex $x_0 \in V$.

In the same way, for $\omega_1, \omega_2 \in \Omega$ with $\omega_1 \neq \omega_2$, let $[\omega_1, \omega_2]$ be the unique geodesic joining ω_1 to ω_2; $[\omega_1, \omega_2]$ is a line, that is, a sequence $\{s_n\}$, $n = 0, \mp 1, \mp 2, \mp 3, \ldots$, of distinct vertices such that $[s_i, s_{i+1}]$ is an edge for every i. Conversely, every line is associated with a pair of ends of X. The reader is referred to [2, 3] for more details.

For $g \in \mathrm{Aut}(X)$, J. Tits has proved in [6] that one and only one of the following holds:

(1) There exists a vertex $v \in V$ such that $g(v) = v$ (in this case g is called a rotation).