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FINITE DIMENSIONAL REPRESENTATION OF CLASSICAL CROSSED-PRODUCT ALGEBRAS

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The paper describes the structure of finite dimensional representations of B_T, the crossed-product algebra of a classical dynamical system $(\alpha_T, \mathbb{Z}, C(X))$ where T is a homeomorphism on a compact space X. The results are used to describe the topology of $\mathrm{Prim}_n(B_T)$ and to partially classify the hyperbolic crossed-product algebras over the torus. One of the main results is that the number of orbits of any fixed length with respect to T is an invariant of B_T. A consequence of that is that the entropy of T is an invariant of B_T, for T a hyperbolic automorphism on the m-torus.

Introduction. The purpose of this paper is to study finite dimensional representations of classical crossed-product algebras. The results are used to describe the primitive ideal space of these algebras and partially classify them. The first two sections deal primarily with finite dimensional representations of B_T, the crossed-product algebra B_T of a classical dynamical system of the form $(\alpha_T, \mathbb{Z}, C(X))$ where T is a homeomorphism on a compact space X. In §1 we study the general form of an irreducible n-dimensional representation of B_T. We show how to adjoin an orbit of length n to each such representation. The idea of adjoining an orbit to each finite dimensional representation is then further explored in §2. We show that the number of connected components in $\mathrm{Prim}_n(B_T)$ is equal to the number of orbits of length n with respect to T. A consequence of this result is that the entropy of T, for T a hyperbolic automorphism on $\mathbf{T}^m$, is an invariant of B_T. In §3 we investigate the classification of the B_T's corresponding to automorphisms on the 2-torus.

Preliminaries. For any integer n we define $E_n\colon B_T \to C(X)$ to be the (continuous) transformation that takes C in B_T to its nth "Fourier" coefficient f_n, see [**1**] for details. Symbolically, we write each C in B_T as $\sum f_n U^n$ where $f_n = E_n(C)$. Let $(\hat{\alpha}, \mathbf{T}, B_T)$ be the C^*-dynamical system defined by the dual action $\hat{\alpha}_\lambda(C) = \sum \lambda^n U^n$, [**2**]. It is known that the Fejer sums of the function $\lambda \to \hat{\alpha}_\lambda(C)$ converge uniformly to