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UNIQUENESS PROBLEM WITHOUT MULTIPLICITIES IN VALUE DISTRIBUTION THEORY

SHANYU JI

Let $H_1, \ldots, H_k$ be hyperplanes in general position in $\mathbf{P}^m$ with $m \geq 2$. Let $A_1, \ldots, A_k$ be pure $(n-1)$-dimensional analytic subsets of $\mathbf{C}^n$ with $\operatorname{codim} A_i \cap A_j \geq 2$ whenever $i \neq j$. Then any linearly non-degenerate meromorphic maps $f, g, h\colon \mathbf{C}^n \to \mathbf{P}^m$ with $f|A_j = g|A_j = h|A_j$ and with $f^{-1}(H_j) = g^{-1}(H_j) = h^{-1}(H_j) = A_j$ for $j = 1, \ldots, k$ satisfy Property (P) if $k = 3m+1$. Consequently such f, g, h are algebraically dependent. If even $n \geq \operatorname{rank} f = \operatorname{rank} g = \operatorname{rank} h = m$, then $k = m+3$ suffices.

1. Introduction. Since Pólya's work [P1] in 1929, the uniqueness problem in value distribution theory has been studied by Nevanlinna [N1], Cartan [C1, C2], Fujimoto [F1, F2], Schmid [S1], Smiley [S5, S6], Carlson [D2], Drouilhet [D1, D2] and Stoll [S11]. One of the main results, given by Fujimoto in 1979 [F2], is that if H_j are hyperplanes in $\mathbf{P}^m$ in general position and v_j are divisors on $\mathbf{C}^n$ whose supports have no common irreducible components for $j = 1, \ldots, m+2$ and if $\mathscr{W}$ is the set of meromorphic maps $f\colon \mathbf{C}^n \to \mathbf{P}^m$ with $f^*(H_j) = v_j$ for $j = 1, \ldots, m+2$, ($f^*(H_j)$ is the pull-back of the divisor of H_j on $\mathbf{P}^m$ by f), then $\mathscr{W}$ cannot contain more than $m+1$ algebraically independent maps. This theorem is in fact a generalization of the Cartan-Nevanlinna theorem (i.e., take $n = m = 1$ and replace "algebraically dependent maps" by "maps" in the above theorem) in 1928.

In this paper, we shall give some analogous results which are without multiplicities. For this kind of problem, Cartan declared [C2] that there are at most two meromorphic functions f, g on $\mathbf{C}$ such that $f^{-1}(a_j) = g^{-1}(a_j)$ for four distinct $a_j \in \mathbf{P}^1$. Cartan's proof appears to have a gap. But some of his original ideas are used in this paper. We also need to use some of Shiffman's and Drouilhet's results [S3], [D2].

Let $H_1, \ldots, H_k$ be hyperplanes in general position in $\mathbf{P}^m$ given by

$$(1.1) \qquad a_0^{(j)} w_0 + \cdots + a_m^{(j)} w_m = 0$$

for $j = 1, \ldots, k$. Let $A_1, \ldots, A_k$ be pure $(n-1)$-dimensional analytic subsets of $\mathbf{C}^n$ with $\operatorname{codim} A_i \cap A_j \geq 2$ whenever $i \neq j$. Put