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1-DIMENSIONAL PHENOMENA IN CELL-LIKE MAPPINGS ON 3-MANIFOLDS

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Two 1-dimensional phenomena are studied. One resides in the 3-manifold domain of a cell-like map $f: M^3 \to Y$ and consists of an infinite 1-skeleton X on which f is 1-1; if, in addition, the nondegeneracy set of f misses a dense subset of each arc in X, then Y admits a natural embedding in $M^3 \times E^1$. The other involves the range; if Y is a 3-manifold except possibly at points of a 1-complex F, topologically embedded in Y as a closed subset, then f can be approximated by another cell-like map $p: M \to Y$ whose nondegeneracy set has embedding dimension ≤ 1 and $f \times \mathrm{Id}: M^3 \times E^1 \to Y \times E^1$ can be approximated by homeomorphisms.

1. Introduction. Consider a proper cell-like surjective mapping $f: M \to Y$ defined on a 3-manifold M. This paper addresses the questions: Under what conditions can f be approximated by a cell-like mapping $F: M \to Y$ for which each set $F^{-1}(y)$ is 1-dimensional? Under what conditions can it be approximated by $F: M \to Y$ such that the nondegeneracy set N_F of F (defined as

$$N_F = \bigcup\{F^{-1}(y) \mid y \in Y \text{ and } F^{-1}(y) \text{ is not a singleton}\})$$

has embedding dimension at most one (in the sense of Štan'ko [Št] and Edwards [E1])?

Several reasons can be adduced for interest in these matters. One simply is to improve known results about which spaces Y are factors of some 4-manifold or, short of that, about which spaces Y have a natural embedding in some 4-manifold (such as in $M \times E^1$). Another reason, part of a personal agenda not completely revealed here, is for use (to put it optimistically) in sought-for internal characterizations of those cell-like images Y that are 3-manifolds, a problem in which map improvement techniques have been exploited with notable success by Edwards [E2].

Before stating the main results, we need certain fundamental definitions. A proper (surjective) map $p: M \to Y$ defined on a manifold M is said to have the *Isotopy Disjoint Arcs Property* (to be abbreviated as: Isotopy DAP) if for each pair of disjoint, locally flat arcs α and β