PACIFIC JOURNAL OF MATHEMATICS
Vol. 135, No. 2, 1988

SPECTRUM AND MULTIPLICITIES FOR RESTRICTIONS OF UNITARY REPRESENTATIONS IN NILPOTENT LIE GROUPS

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Let *G* be a connected, simply connected nilpotent Lie group, and let AT be a Lie subgroup. We consider the following question: for *n* <G G^{A}, how does one decompose *U\K* as a direct integral? In his pioneering paper on representations of nilpotent Lie groups, Kirillov gave a qualitative description; our answer here gives the multiplicities of the representations appearing in the direct integral, but is geometric in nature and very much in the spirit of the Kirillov orbit picture.

1. The problem considered here is the dual of the one investigated by us and G. Grelaud in [2]: give a formula for the direct integral decomposition of Ind^ *o, a* € K^{A}. The answer, too, can be regarded as the dual of the answer in [2]. Let g, *t* be the Lie algebras of *G, K* respectively, and let *g*, t** be the respective (vector space) duals; *P: g** —• 6* denotes the natural projection. Given *n* €ˆ*G,* we want to write

$$n\backslash_{KC} \pm \overset{re}{{}_{\prime}I} \quad n\{a)odv(o);$$

we need to describe *n{o)* and *v*. To this end, we review some aspects of Kirillov theory. In [7], Pukanszky showed that *V* can be partitioned into "layers" U_e, each Ad*(AT)-stable, such that on U_e the *Ad*(K)*-orbits are parametrized by a Zariski-open subset ~L_e of an algebraic variety. (See also §2 of [2].) We can thus parametrize $\hat{K}$ by the union of the *L_e. Let $@_n$ c *g** be the Kirillov orbit corresponding to *n.* There is a unique *e* such that <f_n*nP*~^{l} (U_e) is Zariski-open in <$?_n$. Let £* C S_e be the set of /' *e* $T._e$ such that $P\{@_n)$ meets *K* • /'. It turns out that 27 is a finite disjoint union of manifolds. Let *k** be the maximal dimension of these manifolds; define *v* to be A:*-dimensional measure on the manifolds of maximum dimension and 0 elsewhere. Then we will have

$$\pi|_K \simeq \int_{\Sigma^\pi}^{\oplus} n(l')\sigma_{l'}\, d\nu(l'),$$

where a> corresponds to /' G 27 via the Kirillov orbit picture.