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THE KREIN-MILMAN PROPERTY AND A MARTINGALE COORDINATIZATION OF CERTAIN NON-DENTABLE CONVEX SETS

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The concepts of (strong) martingale representations and coordinatizations are defined, and the notion of a well-separated bush is crystallized. It is proved that if $\mathscr{B}$ is a well-separated uniformly bounded bush such that $\mathscr{B}$ is a strong martingale representation for its closed convex hull W, then W contains no extreme points. It is moreover proved that if K is a closed bounded convex subset of a Banach space with an unconditional skipped-blocking decomposition, then K contains such a bush provided K fails the point of continuity property. This yields the earlier result, due to the authors (unpublished) and to W. Schachermayer, that for closed bounded convex subsets of a Banach space with an unconditional basis, the Krein-Milman property implies the point of continuity property.

1. Let X be a Banach space and C a closed convex subset of X. C is said to have the *Krein-Milman property* (the KMP) if every closed bounded convex subset K of C is the norm-closed convex hull of its extreme points; C is said to have the *point of continuity property* (the PCP) provided every non-empty closed bounded subset K of C has a weak-to-norm point of continuity (a PC) relative to K; C satisfies the *Radon-Nikodým property* (the RNP) if and only if all closed bounded convex subsets K of C are dentable. For $\varepsilon > 0$, say that K is *ε-dentable* if K has a slice of diameter less than ε. A *slice* S of K is a subset of K of the form

$$S = S(f, \alpha, K) = \{x \in K | f(x) \geq \sup f(K) - \alpha\}$$

for some $f \in X^*$, $f \neq 0$ and $\alpha > 0$. K is *dentable* if it is ε-dentable for all $\varepsilon > 0$.

It is well-known that the RNP implies the PCP as well as the KMP (cf. [**BR**], [**DU**] and [**Bo**]). The converse to the first implication is known to be false (cf. [**BR**]); the validity of the converse to the second remains as a fundamental open question.

We introduce here the notions of (strong) martingale representations and coordinatizations in order to investigate the structure of