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SPACES OF CONSTANT PARA-HOLOMORPHIC SECTIONAL CURVATURE

P. M. GADEA AND A. MONTESINOS AMILIBIA

We consider the sectional curvatures for metric ($J^4 = 1$)-manifolds, and study particularly the general expression of the metric and almost-product structure in normal coordinates for para-Kaehlerian manifolds of constant para-holomorphic sectional curvature. We also introduce models of such spaces.

1. Introduction. A *metric* ($J^4 = 1$)*-manifold* (cfr. [3], [11]) is a pseudo-Riemannian manifold (M^n, g) together with a $(1, 1)$ tensor field J such that $J^4 = 1$ and whose characteristic polynomial is $(x-1)^{r_1}(x+1)^{r_2}(x^2+1)^s$ with $r_1 + r_2 + 2s = n$; also, the tensor fields g and J are related by one of the following relations:

(i) $g(JX, Y) + g(X, JY) = 0$ (then g is necessarily pseudo-Riemannian and $r_1 = r_2$);

(ii) g is Riemannian and $g(JX, JY) = g(X, Y)$.

In the first case it is said that g is an aem (adapted in the electromagnetic sense metric), because this situation generalizes in a sense that of Mishra [8] and Hlavatý [4]; in the second one, g is called arm (adapted Riemannian metric).

In this note we consider, g being an aem, the J-Kaehler manifolds, that is ($J^4 = 1$)-manifolds such that $\nabla J = 0$, where ∇ is the Levi-Civita connection of g, and study the J-sectional curvature which generalizes the usual holomorphic-type sectional curvatures. We define the spaces of constant J-sectional curvature, and prove a lemma of Schur type. Also, we obtain explicitly the models corresponding to the situation of an aem g and $J^2 = 1$.

2. Terminology. We shall use the following terminology:

($J^4 = 1$)*-manifold*: the pair (M^n, J), where J is a $(1, 1)$ tensor field such that $J^4 = 1$ and whose characteristic polynomial is $(x-1)^{r_1}(x+1)^{r_2}(x^2+1)^s$ with $r_1 + r_2 + 2s = n$.

e-metric ($J^4 = 1$)*-manifold*: a ($J^4 = 1$)-manifold (M^n, J) together with an aem, that is a pseudo-Riemannian metric g such that $g(JX, Y) + g(X, JY) = 0$.