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FINITE PERMUTATION GROUPS WITH LARGE ABELIAN QUOTIENTS

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We show that if G is a group of permutations on a set of n points and if $|G/G'|$ denotes the order of its largest abelian quotient, then either $|G/G'| = 1$ or there is a prime p dividing $|G/G'|$ such that $|G/G'| \leq p^{n/p}$. Equality holds if and only if G is a p-group which is the direct product of its transitive constituents, with each of those having order p, except when $p = 2$ in which case one must also allow as transitive constituents the groups of order 4, the dihedral group of order 8 and degree 4, and the extraspecial group of order 32 and degree 8.

1. Introduction. In this paper we obtain upper bounds on the orders of abelian quotients of permutation groups in terms of the degrees of the groups, and identify the groups which attain these bounds. First we consider abelian p-quotients for a given prime p.

Recall that a constituent of a permutation group is the restriction of the group to some union of orbits, the restrictions to single orbits being the transitive constituents. By a *transitive non-p'-constituent* we mean a transitive constituent whose order is divisible by p. The *largest p'-constituent* is the restriction to the union of those orbits (if any) on which the group acts as a p'-group.

THEOREM. *If G is a group of permutations on a finite set and if kp denotes the number of points moved by a Sylow p-subgroup of G, then the largest abelian p-quotient of G has order at most p^k. This maximum is achieved by G if and only if G is the direct product of its largest p'-constituent (if any) and of its transitive non-p'-constituents, each of the latter being from the following list of groups:*

(i) C_p, *of order and degree* p;
(ii) C_4, $C_2 \times C_2$, *and* D_8, *of degree* 4, *and the central product* $D_8 \curlyvee D_8$ *of order* 32 *and degree* 8, *when* $p = 2$;
(iii) *the affine groups* AGL(1,3) *and* AGL(1,5), *when* $p = 2$;
(iv) *the affine group* AGL$(1, p + 1)$, *when* $p + 1$ *is a power of* 2.