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UNIQUENESS IN A DOUBLY CHARACTERISTIC CAUCHY PROBLEM

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This article studies uniqueness, in the class of distributions, of solutions of the Cauchy problem for a class of degenerate hyperbolic second order equations, when the initial curve contains a doubly characteristic point. The techniques employed are Carleman estimates and the concatenation method.

1. Introduction. This work is concerned with the uniqueness in the characteristic Cauchy problem for operators with double characteristics at a point of the initial curve.

We obtain an extension of results in **[T2]** and **[BP]**: these authors study uniqueness across $y = 0$ for the operator

$$(D_x - xD_y)(D_x + xD_y) - cD_y;$$

throughout the article we use the notation $D_x = \partial/\partial x$, $D_y = \partial/\partial y$.

Our work has an overlapping with **[N]** and **[K]**. In **[N]**, the study is made in the context of hyperfunction theory and results are proven for operators, e.g., of the type: $(D_x - x^k D_y)(D_x + x^k D_y) - cx^{k-1}D_y$, k a natural number; our method of proof uses only the theory of distributions. In **[K]**, operators, e.g., like

$$P(a, b) = [D_x - a(x)D_y][D_x + a(x)D_y] + b(x)D_y,$$

where a has a zero of order one at zero, are dealt with; in our work, if, say, b is non-negative then a is allowed to vanish to an arbitrary odd order k at zero.

Section 2 contains the proof of the Carleman estimates which yield the uniqueness across $y = 0$, in the class C^2, for the operator $P(a, b)$, under suitable assumptions. The results of this section are more general than what we needed in our applications. In the beginning of §3, we specialize our operator $P(a, b)$ to the case $a(x) = -ax^k$, $b(x) = -cx^{k-1}$, k odd, and, by using the concatenations in **[GT]**, we prove uniqueness, in the class C^m, where m depends on c and c avoids a certain sequence of real numbers. When c takes on such values, it is possible to prove that there is non-uniqueness, even in the class C^∞