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REGULARIZATION OF ACTIONS OF GROUPS AND GROUPOIDS ON MEASURED EQUIVALENCE RELATIONS

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Dedicated to H. A. Dye

The paper deals with the regularization problem for locally compact groups of non-strict automorphisms of measured equivalence relations. It is shown that by means of inessential reorganization of the equivalence relation and the group action one can make all the automorphisms of the given action to be strict with respect to the equivalence relation. A similar problem is solved for an action of a measure groupoid which leaves invariant mod 0 **a measured equivalence relation.**

1. The study of full groups introduced by H. A. Dye [**5**] as well as a consideration of outer conjugacy of subgroups of the full group normalizer (see the paper by A. Connes and W. Krieger [**4**] and the subsequent works [**1**], [**2**], [**3**], [**8**]) forces one to deal with maps and families of transformations that behave properly only almost everywhere. This as a rule does not cause any difficulties when the transformation groups in question are nonsingular and countable. One needs only to discard a null set in order to make the action of such a group regular. However, in the case of continuous transformation groups this method of regularization is inappropriate in general.

The first step in studying the regularization of actions of continuous groups was made by G. Mackey in [**10**], where the existence and uniqueness of a point realization for an action of a locally compact group G as automorphisms of a Boolean algebra were established. In order to form the point realization of a Boolean G-space, the author applies the properties of a universal G-space on which the G-action is regular. This approach got its complete basis later in works of A. Ramsay [**12**], [**14**]. Similar problems were considered by A. M. Vershik [**15**] with another technique used.

The present paper contains a solution of the regularization problem for groups of non-strict automorphisms of measured equivalence relations based on results of [**12**], [**14**] (§2). It is shown that by means of inessential reorganization of the equivalence relation and the group