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ON THE VANISHING OF $H^n(\mathscr{A}, \mathscr{A}^*)$ FOR CERTAIN C^*-ALGEBRAS

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Dedicated to the memory of Henry A. Dye

The norm continuous Hochschild cohomology for a C^*-algebra $\mathscr{A}$ with coefficients in the dual space vanishes if either $\mathscr{A}$ is nuclear or $\mathscr{A}$ has no bounded traces. The norm continuous cyclic cohomology for a C^*-algebra with no bounded traces vanishes.

1. Introduction. There has been some success in computing Hochschild cohomology groups for von Neumann algebras, especially the result that any derivation on a von Neumann algebra is inner [**11**, **14**]. We also have the results by Johnson, Kadison and Ringrose [**10**] which through the work of Connes [**4**] can be phrased: The Hochschild cohomology for an injective von Neumann algebra with coefficients in a dual normal module vanishes. Conversely Connes has proved [**5**], that this property actually characterises the injective von Neumann algebras. Recently E. G. Effros and the present authors have computed some cohomology groups and shown that the completely bounded cohomology vanishes if the module is an injective von Neumann algebra which contains the algebra in question. If the algebra is a C^*-algebra and the coefficients come from another C^*-algebra, very little is known in general, and it is clear that in this case the bounded cohomology will not vanish unless the algebra is very "nice" [**8**]. In the present paper we will prove that the norm continuous Hochschild cohomology for a C^*-algebra $\mathscr{A}$ with coefficients in the dual space $\mathscr{A}^*$ does vanish, if $\mathscr{A}$ is nuclear or if $\mathscr{A}$ has no bounded traces. The result for nuclear C^*-algebras is not new, in the sense that it has been known to a number of people. It follows relatively easily from the fact, that the double dual of a nuclear C^*-algebra is an injective von Neumann algebra [**2**]. The result for infinite C^*-algebras is proved by methods which are closely related to the techniques developed by Johnson, Kadison and Ringrose [**10**] in order to reduce the norm continuous cohomology to the ultraweakly continuous cohomology. Their results do not fit exactly because $\mathscr{A}^*$ is not a dual normal module for the von Neumann algebra $\mathscr{A}^{**}$. Despite this a modification of well-known techniques