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ALGEBRAIC CHARACTERIZATION OF THE VACUUM FOR QUANTIZED FIELDS TRANSFORMING NON-UNITARILY

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Dedicated to the memory of Henry A. Dye

The vacuum as an expectation value form on the Clifford or Weyl algebra over an orthogonal or symplectic real linear space, invariant under a given group of automorphisms of such, is treated without assumptions as to self-adjointness or positivity. This is necessary for the quantization of fields that transform non-unitarily, in particular indecomposably, such as the full section spaces of typical conformally invariant bundles over space-times. A stability condition in the nature of positivity of the energy is shown to be sufficient to characterize the vacuum for the basic case of a one-parameter group. In application e.g. to spannor fields transforming under $SU(2,2)$, this results in a vacuum invariant under the maximal subgroup K, giving rise to a natural broken symmetry.

1. Introduction. From early heuristic beginnings, quantization has developed into a vast enterprise that is in large part rigorously mathematical. The finite-dimensional case in standard quantum mechanics was settled essentially by Stone and van Neumann, whose work paid important mathematical dividends. The infinite-dimensional case was more refractory, but there are now simple and natural characterizations of the free boson and fermion quantum fields. At first the quantization procedure treated only systems whose single-particle structure transformed unitarily under the basic symmetry group, as in the most familiar physical cases (e.g. the Klein-Gordon, Maxwell, and Dirac equations). The associated unitary action on the quantized field Hilbert space was then essentially the direct sum of the symmetrized or skew-symmetrized tensor powers of the single-particle representation, depending on the "statistics" of the field, i.e. whether bosons or fermions were involved.

In practice, the single-particle action was often, in the more interesting cases, such as when interactions were involved, not unitary or at least not manifestly unitary, but only symplectic in the case of bosons or orthogonal in the case of fermions. This led to the study of the question on unitary implementability on the quantum field of