PACIFIC JOURNAL OF MATHEMATICS
Vol. 138, No. 1, 1989

APPROXIMATE INVERSE SYSTEMS OF COMPACTA AND COVERING DIMENSION

SIBE MARDEŠIĆ AND LEONARD R. RUBIN

Approximate inverse systems of metric compacta are introduced and studied. The bonding maps in these systems commute only up to certain controlled values. With every such system $\mathbf{X} = (X_a, \varepsilon_a, p_{aa'}, A)$ are associated a limit space X and projections $p_a: X \to X_a$. A compact Hausdorff space X has covering dimension $\dim X \leq n$ if and only if it can be obtained as the limit of an approximate inverse system of compact polyhedra of dimension $\leq n$. The analogous statement for usual inverse systems is known to be false.

1. Introduction. An inverse system of spaces $\mathbf{X} = (X_a, p_{aa'}, A)$, in the usual sense, consists of a directed set A, spaces X_a, $a \in A$, and maps $p_{aa'}: X_{a'} \to X_a$, $a \leq a'$, such that $p_{aa} = \text{id}$, $p_{aa'}p_{a'a''} = p_{aa''}$, $a \leq a' \leq a''$. The (usual) inverse limit of X is the subspace $X \subseteq \prod X_a$ which consists of all points $x = (x_a) \in \prod X_a$ such that $p_{aa'}(x_{a'}) = x_a$ whenever $a \leq a'$. Projections $p_a: X \to X_a$ are just the restrictions $p_a = \pi_a|X$ of the projections $\pi_a: \prod X_a \to X_a$.

It is well known that the inverse limit of an inverse system of non-empty compact spaces X_a is a non-emepty compact space X. If the covering dimension $\dim X_a \leq n$, $a \in A$, then also $\dim X \leq n$. In particular, a limit of compact polyhedra P_a with $\dim P_a \leq n$ is a compact Hausdorff space with $\dim X \leq n$.

On the other hand, every compact Hausdorff space X is the limit of an inverse system of compact polyhedra P_a [**1**]. If X is a compact metric space and $\dim X \leq n$, one can obtain X as the limit of an inverse sequence of compact polyhedra P_a with $\dim P_a \leq n$ ([**2**], also see [**4**]). However, the analogous statement for compact Hausdorff spaces is false as shown in 1958 independently by S. Mardešić [**4**] and B. A. Pasynkov [**6**]. These authors produced examples of compact Hausdorff spaces X with $\dim X = 1$ which cannot be represented as inverse limits of inverse systems of compact polyhedra of dimension ≤ 1. Further examples of this type were given by Mardešić in [**3**] and Pasynkov in [**7**]. Recently, Mardešić and T. Watanabe [**5**] have shown that a 1-dimensional compact Hausdorff space considered by