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RANGE TRANSFORMATIONS ON A BANACH FUNCTION ALGEBRA. II

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Dedicated to Professor Junzo Wada on his 60th birthday

In this paper, localization for ultraseparability is introduced and a local version of Bernard's lemma is proven. By using these results it is shown that a function in $\mathrm{Op}(I_D, \operatorname{Re} A)$ is harmonic near the origin for a uniformly closed subalgebra A of $C_0(Y)$ and an ideal I of A unless the uniform closure $\operatorname{cl} I$ of I is self-adjoint; in particular, it is shown that $\operatorname{cl} I$ is self-adjoint if $\operatorname{Re} I \cdot \operatorname{Re} I \subset \operatorname{Re} A$, which is not true when I is merely a closed subalgebra of A.

1. Introduction. Let Y be a locally compact Hausdorff space, and $C_0(Y)$ (resp. $C_{0,R}(Y)$) be the Banach algebra of all complex (resp. real) valued continuous functions on Y which vanish at infinity. If Y is compact, we write $C(Y)$ and $C_R(Y)$ instead of $C_0(Y)$ and $C_{0,R}(Y)$ respectively. Thus $C(Y)$ (resp. $C_R(Y)$) is the algebra of all complex (resp. real) valued continuous functions on Y if Y is compact. For a function f in $C_0(Y)$, $\|f\|_\infty$ denotes the supremum norm on Y. We say that A is a Banach algebra (resp. space) included in $C_0(Y)$ with the norm $\|\cdot\|_A$ if A is a complex subalgebra (resp. space) of $C_0(Y)$ which is a complex Banach algebra (resp. space) with respect to the norm $\|\cdot\|_A$ (resp. such that $\|f\|_\infty \leq \|f\|_A$ holds for every f in A). It is well known that the inequality $\|f\|_\infty \leq \|f\|_A$ holds for every f in a Banach algebra A included in $C_0(Y)$ with the norm $\|\cdot\|_A$. Thus we may suppose that a Banach algebra included in $C_0(Y)$ is a Banach space included in $C_0(Y)$. We say that E is a real Banach space included in $C_{0,R}(Y)$ with the norm $\|\cdot\|_E$ if E is a real subspace of $C_{0,R}(Y)$ which is a real Banach space with respect to the norm $\|\cdot\|_E$ such that $\|u\|_\infty \leq \|u\|_E$ holds for every u in E. A (resp. real) Banach space or algebra included in $C_0(Y)$ (resp. $C_{0,R}(Y)$) is said to be trivial if it coincides with $C_0(Y)$ (resp. $C_{0,R}(Y)$).

If A is a Banach space included in $C_0(Y)$ with the norm $\|\cdot\|_A$ for a locally compact Hausdorff space Y, $\operatorname{Re} A = \{u \in C_{0,R}(Y) : \exists v \in C_{0,R}(Y)$ such that $u + iv \in A\}$ is a real Banach space with respect to