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ON CONSTRUCTIONS SIMILAR TO THE BURNSIDE RING FOR COMMUTATIVE RINGS AND PROFINITE GROUPS

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The question of finding all isomorphism classes of finite dimensional commutative semisimple rational algebras is an unsolved one and is equivalent to the question of finding all number fields. We feel that this problem may eventually be solved by the Burnside ring method, where the number fields are related to each other in many different ways. In this note we generalize the problem to the larger setting of *G*-algebras, where *G* is a finite abelian group. This gives even more relations—which we investigate. In order to see what is special about the rationals, we work as long as possible with a commutative ring *R*.

Separable ring extensions of a commutative ring R correspond by Galois theory to actions of a profinite group Γ. Hence we work with either commutative rings or profinite groups. In either case, we use a "twisting" by a finite abelian group G (more specifically, G cyclic of prime order). This allows us to form what we call the $*$-product. In the ring case, the $*$-product of R-algebras A and B is the R-algebra

$$A *_G B = \Big\{\sum x_i \otimes y_i \in A \otimes_R B \mid \sum x_i \otimes y_i = \sum \sigma x_i \otimes \sigma^{-1} y_i \ \forall \sigma \in G\Big\}.$$

One more parameter J is needed to get additive inverses in the resulting commutative ring with identity. (The addition comes from direct product in the ring case and from disjoint union in the case of sets with action of the profinite group.) In the ring case, we call the resulting ring $W(R, G; J)$. For k a finite field and G cyclic of order p, we calculate $W(R, G; J)$ explicitly.

1. General theory. Let R be a commutative ring which is nonzero (i.e. $1 \neq 0$) and which has no idempotents except 0 and 1. Let G be a finite abelian group. By an (R, G)-*algebra* (A, θ) we will mean a commutative, finitely generated, projective, separable R-algebra A with a group homomorphism θ from G to $\mathrm{Aut}_R(A)$. Let (A, θ) and