PACIFIC JOURNAL OF MATHEMATICS
Vol. 138, No. 2, 1989

ON THE HARDY SPACE H^1 ON PRODUCTS OF HALF-SPACES

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We show that the Hardy space $H^1_{\text{anal}}(\mathbf{R}^2_+ \times \mathbf{R}^2_+)$ can be identified with the class of functions f such that f and all its double and partial Hilbert transforms $H_k f$ belong to $L^1(\mathbf{R}^2)$. A basic tool used in the proof is the bisubharmonicity of $|F|^q$, where F is a vector field that satisfies a generalized conjugate system of Cauchy-Riemann type.

Introduction. The interest of a theory for the H^p spaces on products of half-spaces was first raised by C. Fefferman and E. M. Stein in the now classic paper "H^p spaces of several variables" [**6**]. Afterward several authors have contributed on this subject. It is worth mentioning the survey paper by C. Y. A. Chang and R. Fefferman [**4**], and the references quoted there. In particular, the H^1 spaces on products of half-spaces was studied by H. Sato [**8**] giving definitions via maximal functions and via the multiple Hilbert transform. On the other hand Merryfield [**7**] proves the equivalence of the definitions given via the area integrals and via the multiple Hilbert transforms. More recently S. Sato [**9**] proved the equivalence between the Lusin area integral and the nontangential maximal function.

The purpose of this paper is to derive directly the equivalence of the definitions of the H^1 space given via the multiple Hilbert transforms and via an L^1 condition on a biharmonic vector field $F = (u_1, u_2, u_3, u_4)$ which is a solution of a generalized Cauchy-Riemann system introduced by Bordin-Fernandez [**3**]. The main tool we shall use is the bi-subharmonicity of $|F|^q, 0 < q < 1$. But the proof of this fact here is different from the classical one given by Stein-Weiss [**10**]. We rely on ideas of A. P. Calderón, R. Coiffman and G. Weiss (see [**5**]). We shall confine ourselves to the bidimensional case.

This paper is part of the author's doctoral thesis presented to UNICAMP in 1982, and the results are announced in [**1**] and [**2**].

NOTATION. We shall use the following notations throughout:

$$\square = \{k = (k_1, k_2),\ k_j = 0, 1,\ j = 1, 2\}$$